Storage Capacity of Letter Recognition in Hopfield Networks

Gang Wei (gwei@cs.dal.ca)
Zheyuan Yu (zyu@cs.dal.ca)

Faculty of Computer Science,
Dalhousie University,
Halifax, N.S.,
Canada B3H 1W5

Abstract: In this paper, the Hopfield neural networks model is discussed and implemented for letter recognition. Hebbian and pseudo-inverse learning rules are applied. Comparisons are made between these two rules. The storage capacity, basin of attractions are studied.

Keywords: *auto-associative memory, Hopfield neural networks, Hebbian, pseudo-inverse, learning rule, storage capacity, basin of attraction.*

1 INTRODUCTION

Associative memory is a dynamical system which has a number of stable states with a domain of attraction around them [1]. If the system starts at any state in the domain, it will converge to the locally stable state, which is called an attractor. In 1982, Hopfield [2] proposed a fully connected neural network model of associative memory in which patterns can be stored by distributed among neurons, and we can retrieve one of the previously presented patterns from an example which is similar to, or a noisy version of it.

The Hopfild model consists of *N* neurons and N^2 synapses. Each neuron can be in one of two states. It is based on the difference equation

$$s_i(t+1) = sign[\sum_j w_{ij} s_j(t)]$$

Where $s_i(t)$ is the state of neuron *i*. w_{ij} is the synaptic strength from neuron *j* to *i*. The appropriate determined synaptic weights can store patterns as fix points. The previously stored patterns can be retrieved from the representation of partial patterns.

The dynamical behavior of its neuron states strongly depends on synaptic strength between neurons. The specification of the synaptic weights is conventionally referred to as *learning*. Hopfield used the Hebbian learning rule [3] to determine weights. Since Hopfield's proposal, many alternative algorithms for learning and associative recalling have been proposed to improve the performance of the Hopfield networks. We will discuss the Hebbian rule and pseudo-inverse rule and apply them to letter recognition. The comparisons are made between these two rules.

2 Different learning rules

Hebbian rule

The Hebbian learning rule is given by **:**

$$w_{ij}^{v} = 0 \;\; \forall i, j \;\; \text{and} \;\; w_{ij}^{v} = w_{ij}^{v-1} \frac{1}{N} \sum_{u} \xi_i^v \xi_j^v$$

Where w^v is the state of weight matrix after the *v*th patters have been learnt but before the (*v*+1)th pattern has been introduced, ξ^v is the patterns for the *v* steps, where N is the number of neurons. The Hebbian rule is local and incremental, but has a low absolute storage capacity of $\frac{n}{2\ln n}$. This capacity decreases significantly if patterns are correlated. Its performance is poor in terms of storage capacity, attraction, and spurious memories [4].

Pseudo-inverse Rule

The pseudo-inverse rue is given by

$$w_{ij}^{v} = \frac{1}{N} \sum_{v=1}^{m} \sum_{u=1}^{m} \xi_i^v (Q^{-1})^{vu} \xi_j^u$$

Where $Q = \frac{1}{N} \sum_{k=1}^{n} \xi_k^v \xi_k^u$, and m is the total number of patterns with m<n, *N* is the number of neurons.

The pseudo-inverse (PI) learning rule (LR), which is also called the projection learning rule [5, 6]. PI LR is accredited for its high retrieval capability. The limit of 0.5*N* for the associative capacity, obtained by Personnaz *et al*., Kaner and Sompolinsky [5, 7]. For comparison, Hebbian correlation LR retrieves only up to 14%*N* prototypes.

However, the pseudo-inverse rule is not local or incremental, because it involves the calculation of an inverse.

3 Storage Capacity

By using computer simulation, Hopfield suggested that if a small error in recalling is allowed, the maximum number *p* of the patterns to be stored in a network with *N* neurons is 0.15*N*, Later, it was showed theoretically that the storage capacity is *p*=0.138*N* by Amit et al. [8] by using *replica* method. The limitation is attributed to the fact that the network is trapped to the so called spurious local minima. In 1987 McEliece et al. [9] proved that when $p < N / 4 \ln N$ holds, the Hopfield's model is able to recall all the memorized patterns without error.

Other learning schemes have been proposed to escape the local minima and increase the storage capacity. Kohonen et al. [10] proposed a new learning mechanism for non-zero self-connection networks called *pseudo-inverse* learning rule by extending the Hebbian learning rule, and enlarged the capacity to $p = N$. Gardner [11] showed that the ultimate capacity will be $p = 2N$ as the basin size tend to zero.

4 EXPERIMENT

All experiments in this paper are carried out on the model of letter patterns. The 26 letter patterns are presented with binary representation (0 or 1) of 156 neurons. The synaptic weight values are explored with Hebbian rule and pseudo-inverse rule. The update scheme is based exactly on the Hopfield model. In the stage of retrieval, 10% noise is applied to letter patterns. Noise is created by randomly inverting pixels of a pattern. We trained the networks with increasing number of letter patterns, which are randomly chosen from 26 letter patterns, and the difference is recorded between the final state of the networks and the corresponding training patterns for 30 repeats.

5 RESULTS AND DISCUSSION

5.1 Storage Capacity

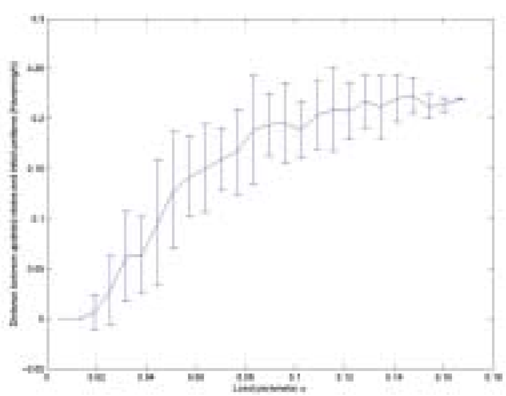

Figure 1 Load parameters Vs average Hamming distance between updated states and initial states (Hamming/n) for Hebbian rule.

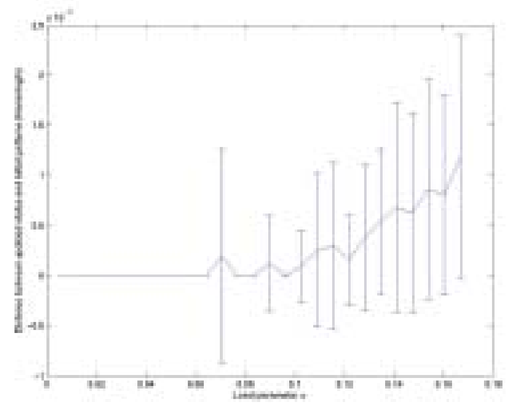

Figure 2 Load parameters Vs average Hamming distance between updated states and initial states (Hamming/n) for pseudo-inverse rule.

Figure 1 and 2 show the storage capacity for Hebbian rule and pseudo-inverse rule. The trained patterns are initialized with 10% noise. The average distance is measured by the average Hamming distance between updated states and initial states divided by the number of neurons n=156 for 30 repeat tests. Figure 1 shows that the storage capacity for Hebbian rule, and figure 2 for pseudo-inverse rule. The load parameter is define as $\alpha = N^{pat} / C$, where N^{pat} is the number of trained pattern, C is the number of nodes in the fully connected networks.

According to figure 1 and 2, the storage capacity of Hebbian rule is about 0.012. That means 2 letter patterns can be stored and retrieved without error by the model. And the storage capacity of pseudo-inverse rule is about 0.064. That means 10 letter patterns can be stored and retrieved without error by the model.

Also we notice that the average distance between updated states and initial states for pseudo-inverse rule is much smaller than that for Hebbian rule

The storage capacity of our Hopfield networks, for Hebbian rule is 0.012 and for psedo-inverse rule is 0.064, are far away from the result in theory which are 0.138 and 1. This can be caused by the following factors [12].

1) A high correlation of the training patterns. Correlations between the patterns worsen the performance of the network. Pseudo-inverse rule is one of the solutions to lower the correlation of the training patterns by preprocesse the patterns.

2) The capacities of Hopfield networks are related with sparse representation of training patterns.

3) Global inhibition is another factor that can affect storage capacity.

5.2 Basin of Attraction

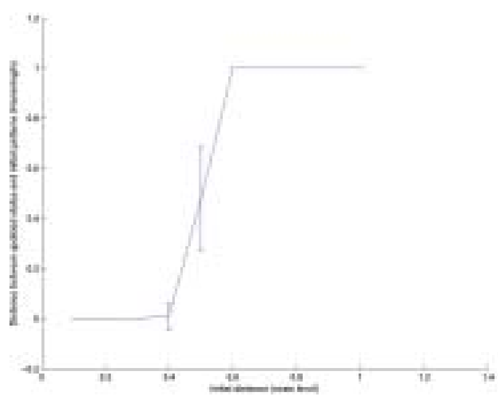

Figure 3 Initial distance (noise level) Vs average Hamming distance between updated states and initial states (Hamming/n) for Hebbian rule.

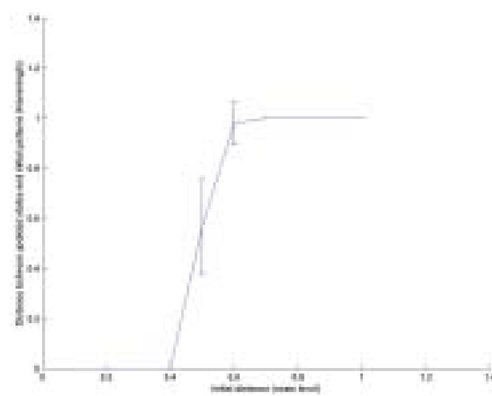

Figure 4 Initial distance (noise level) Vs average Hamming distance between updated states and initial states (Hamming/n) for pseudo-inverse rule

Figure 3 and 4 show that the initial distance (noise level) Vs average Hamming distance between updated states and initial states (Hamming/n) for Hebbian rule and pseudo-inverse rule respectively. The distance is measured by Hamming distance between updated states and initial states divided by number of neurons n=156. 2 patterns are chosen randomly from 26 letter patterns and presented to the network for each test, the first trained pattern is presented and the distance is recorded.

We can see from figure 3 and 4 that the difference is not significant between the attraction basins for Hebbian rule and that for pseudo-inverse rule at a very low loading.

The pattern completion ability of the associative memory makes the trained patterns point attractors of the network. Figure 3 and 4 demonstrate that, for both Hebbian and pseudo-inverse rules, randomly chosen letter pattern converge to desired attractor if the initial distance is less than a certain value d, which is the size of basin of the attraction.

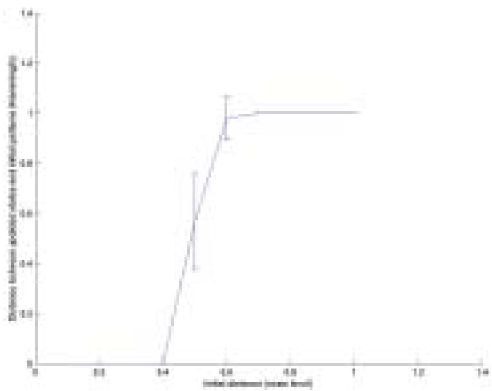

Figure 5 Store 2 patterns and retrieve first pattern for pseudo-inverse rule

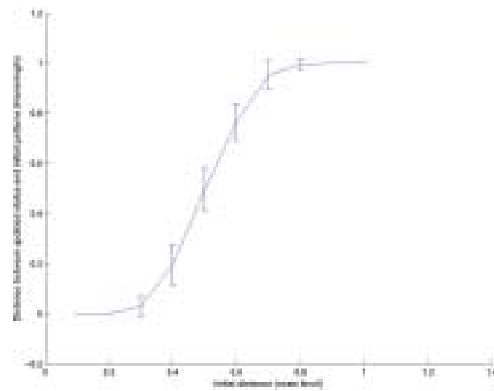

Figure 6 Store 15 letter patterns and retrieve first pattern for pseudo-inverse rule

Figure 5 and 6 are similar to figure 3 and 4 but for only store 2 patterns and 15 letter patterns with pseudo-inverse rule respectively. The retrieving of the first letter pattern is recorded.

It shows if the networks only store 2 letter patterns, the networks can retrieve the letter pattern without error with the initial distance as long as 0.4. However, if the networks store 15 letter patterns, the longest initial distance decrease to 0.2. That means the size of basin of the attraction will became smaller with more patterns are stored.

6 CONCLUSIONS

We have discussed Hopfield model of associative memory and implemented a Hopfield networks for letter recognition. We found that the networks with 156 neurons would store and retrieve without error for only 2 patterns with the Hebbian learning rule in average. The network of this size using pseudo-inverse rule can store an average of 10 patterns and retrieve without error. Also we found that the size of basin of the attraction of the Hopfield networks will became smaller with more stored patterns.

REFERENCES

[1] J.Komlos, and R. Paturi. Convergence Results in an Associative Memory Model. *Neural Networks 1*. pp239-250, 1988.

[2] J. J. Hopfield. Neural Networks and Physical System with Emergent Collective Computational Abilities. *Proceedings of the National Academy of Sciences, USA. 79*. pp2554-2558, 1982.

[3] D. O. Hebb. The Organization of Behavior. *Wiley*, 1949.

[4] Storkey A.J. Increasing the capacity of the Hopfield network without sacrificing functionality, *ICANN97* p451-456, 1997

[5] L. Personnaz, I. Guyon and G. Dreyfus. Collective computational properties of neural networks: New learning mechanisms, *Phys. Rev. A* vol 34, pp. 4217-4228, 1986.

[6] D.O. Gorodnichy. A way to improve error correction capability of Hopfield associative memory in the case of saturation, *HELNET 94-95 International Workshop on Neural Networks Proceedings,* vol. I/II, pp. 198-212, VU University Press, Amsterdam, 1996

[7] I. Kanter and H. Sompolinsky. Associative recall of memory without errors, *Phys. Rev. A* vol 35, pp. 380-392, 1987.

[8] D. J. Amit, H. Gutfreund, and H. Sompolinsky. Statistical Mechanics of Neural Networks near Saturation. *Annals. of Physics 173,* pp30-67, 1985.

[9] R. J. McEliced, E. C. Posner, E. R. Rodemick, and S. S. Venkatesh. The capacity of the Hopfield Associative Memory. *IEEE Trans. Information Theory IT-33*. pp461-482, 1987.

[10] T. Kohonen, and M. Ruohonen, Representation of Associated Data by Matrix Operators. *IEEE Trans. Computer C-22(7)*. Pp701-702, 1973.

[11] E. Gardner and B. Derrida. Three unfinished works on the optimal storage capacity of networks. *3. Phys. A: Math. Gen.*, 22:1983-1994, 1989.

[12] Thomas P. Trappenberg Fundamental of computational neuroscience, Oxford University Press, p190-196