Gathering Teams of Deterministic Finite Automata on a Line

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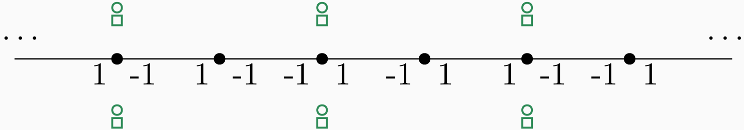
Introduction

- Mobile Agents: Modeled as deterministic finite automata,
- An infinite line: Oriented or unoriented,
- Goal: Gathering and stop.



An Infinite Line

Each node is unlabeled and its ports are labeled by -1 and 1.

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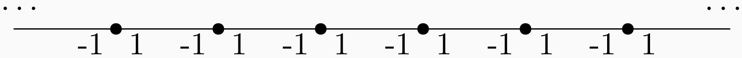

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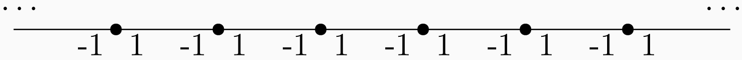

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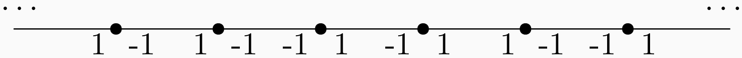

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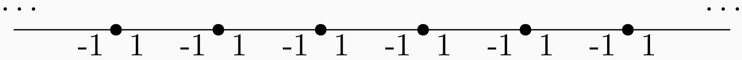

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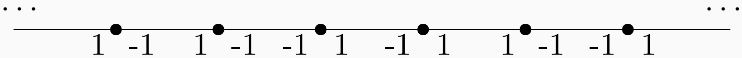
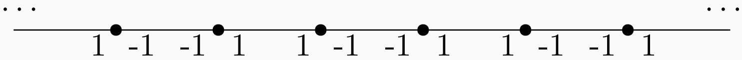


  - Homogeneous Line



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- share the base in each team.
- R, x, L are known by each agent.

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- decides to stay idle, leave from current node via port -1 or port 1.

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- decides the port labeling at each node of an unoriented line

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- Algorithm: If so, design automata that gather all the agents at the same node.

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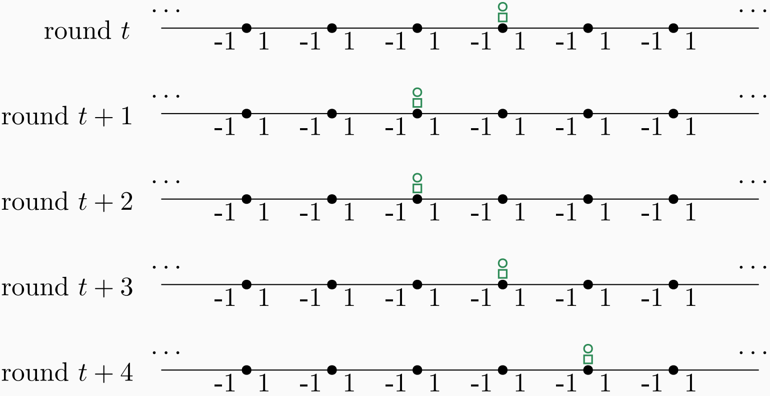
- meetings inside an edge is not allowed, and
- agents cannot "see" other agents at a distance.

Our Results

Team Size	Oriented Line	Unoriented Line
$x = 1$	Infeasible	Infeasible
$x = 2$	$\Theta(D)$	$\Theta(D \log L)$
$x > 2$	$\Theta(D)$	$\Theta(D)$

Preliminary: Trajectories, Periodic and Boundaries

Trajectory: A trajectory of an agent is defined as an infinite sequence of terms drawn from $\{-1, 0, 1\}$.



Rounds	...	t	t+1	t+2	t+3	t+4	...
Trajectories		...	-1	0	1	1	...

Preliminary: Periodic

Proposition

The trajectory of any agent navigating either in the oriented or in the homogeneous line and starting at any node of it is periodic.

-1 -1	1 1 1 -1 -1 -1	1 1 1 -1 -1 -1	1 1 1 -1 -1 -1

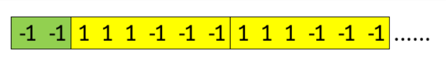
Notes: The adversary can always fool a single mobile agent such that it cannot explore an infinite line by itself.

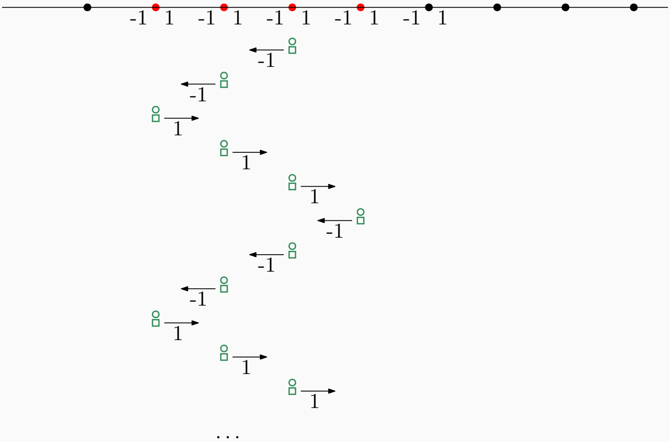
Boundaries of Periodic Trajectories

Three types of periodic trajectories on homogeneous and oriented lines:

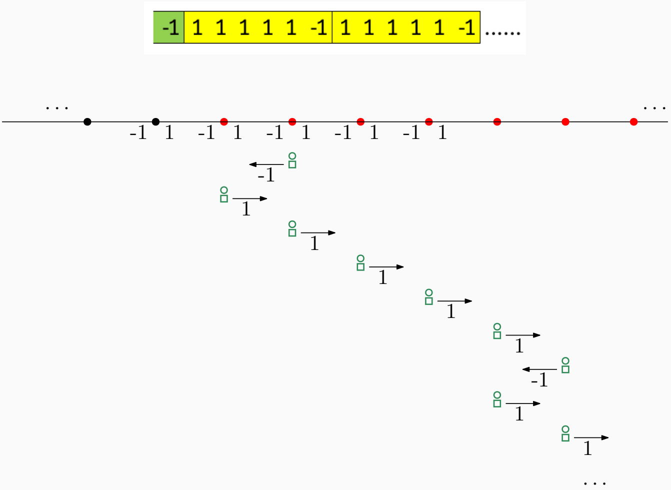
- *minus-progressing*,
- *plus-progressing* and,
- *bounded*.

Bounded trajectories





Minus-Progressing and Plus-Progressing Trajectories



A plus-(or minus-) progressing trajectory is associated with *speed*.

The Lower Bound: Team Size is Two

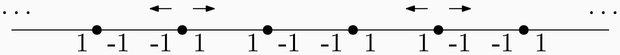
The lower bound result

- On a Homogeneous line
- two teams of size two based at two nodes at a distance D
- gathering takes at least $cD \log L$ rounds, for some constant c

Notes: Consider two teams $(3, 5)$ and $(3, 7)$. Agent with label 3 in these teams might behave differently.

The Lower Bound: Team Size is Two

Canonical teams



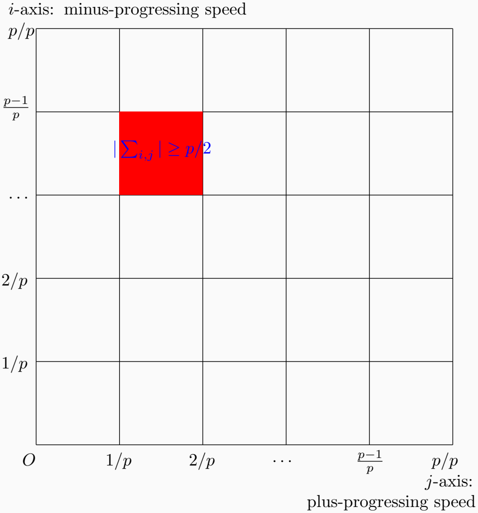
Lemma

There are always at least $(\lfloor L/2 \rfloor - 1)$ *canonical teams that the adversary can choose from; otherwise, the adversary could always avoid gathering.*

E.g., $\{1,2\}$, $\{3,4\}, \{7,8\}, \{9,10\}, \{11,12\}$, ... , $\{2C-1, 2C\}$.

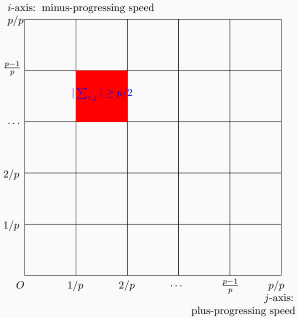
The Lower Bound: Team Size is Two

- Let $p = \lfloor L^{1/3} \rfloor$;
- divide all the canonical teams into p^2 groups as follows:

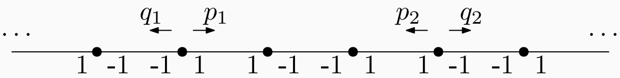


The Lower Bound: Team Size is Two

Lemma: *There exists an instance such that the first meeting happens no later than $c' D \log L$ rounds.*



- W.l.o.g, assume that $i \geq j$,
- there exists agents p_1 and p_2 such that their meeting requires $cD \log |\sum_{i,j}|$ rounds,
- if $i > j$, then $v(q_1) \geq v(p_2)$ and $v(q_2) \geq v(p_1)$,
- otherwise, catch-up cannot happen quickly.



Open Problems

- Gathering of teams of automata in arbitrary (connected) infinite graphs and
- Gathering of teams of possibly different sizes

Thanks!

Appendix

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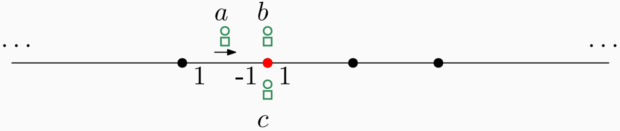

- $\mathcal{I}$:
  - $I_a = \{(L, R)\} \times \{-1\} \times \{Q^a \cup Q^b \cup Q^c\}$,
  - $I_b = \{(L, R)\} \times \{0\} \times \{Q^a \cup Q^b \cup Q^c\}$,
  - $I_b = \{(L, R)\} \times \{0\} \times \{Q^a \cup Q^b \cup Q^c\}$.
- δ:
  - $q_a(\in Q^a) \times I \to q'_a(\in Q^a)$,
  - $q_b(\in Q^b) \times I \to q'_b(\in Q^b)$,
  - $q_c(\in Q^c) \times I \to q'_b(\in Q^c)$
- λ :
  - $q_a \times I_a \to o_a$,
  - $q_b \times I_b \to o_b$,
  - $q_c \times I_c \to o_c$.

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- Simultaneous start:
  - If the start was not simultaneous, then no bound on gathering time could be established

Related Work

Rendezvous and Gathering on graph networks

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- Nodes without labels

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Feasibility: Team Size is One

Theorem

Consider an arbitrary set of agents. Then the adversary can place these agents at distinct nodes of the oriented line in such a way that no pair of agents will ever meet.

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