CSCI 3110 Tutorial 9

Reviewed July 19, 2019

We showed in class how to build a data structure that can support three-sided range reporting. That is, a data structure that can maintain a set S of points in the plane under insertions and deletions and answer queries of the form $R = [l, r] \times [b, \infty)$. This data structure uses an (a, b)-tree on the x-coordinates and stores the y-coordinates as a heap within the search tree. Call this data structure a standard priority search tree.

Recall that insertions and deletions take $\mathrm{O}\left(\lg^2 n\right)$-time and 3-sided range queries take $\mathrm{O}(\lg n + k)$-time. You should also be familiar with the construction and query times for regular (a, b)-trees and d-dimensional range trees.

1. For each of the following type of queries, can the standard priority search tree support the query and how? If it cannot, then describe a data structure that can support the query and how long the query will take.

   (a) $R1 = [l, r] \times [b, \infty)$,

   (b) $R2 = [l, r] \times [-\infty, t)$,

   (c) $R3 = [l, \infty) \times [b, \infty)$,

   (d) $R4 = (-\infty, r] \times [b, \infty)$,

   (e) $R5 = [l, r] \times (-\infty, \infty)$,

   (f) $R6 = [-\infty, \infty] \times (b, t)$, and

   (g) $R7 = [l, r] \times [b, t]$

2. Each of these queries can be classified as 2-sided, 3-sided, or 4-sided range queries. Develop an efficient data structure that can maintain a set S of points and support any 2-sided or 3-sided range query. How long do the queries take, how long does it take to build the data structure statically and dynamically, and how much space does the data structure take?