1. Consider the interval scheduling problem discussed in class. Suppose that instead of always selecting the interval that ends first, we instead select:

   i) A compatible interval uniformly at random,
   ii) The median interval by start time (break ties by earliest end time),
   iii) The shortest interval compatible with all previously selected intervals,
   iv) The compatible interval that overlaps the fewest other remaining intervals, or
   v) The compatible remaining interval which ends last.

Provide a counterexample showing that these algorithms do not always yield an optimal solution.

2. Consider an undirected graph $G = (V, E)$ with distinct nonnegative edge weights $w_e \geq 0$. Suppose that you have computed a minimum spanning tree of $G$, and that you have also computed shortest paths to all nodes from a particular node $s \in V$.

Now suppose each edge weight is doubled: the new weights are $w'_e = 2w_e$.

(a) (10 pts) Does the minimum spanning tree change? Give an example where it changes or prove it cannot change.

(b) (10 pts) Do the shortest paths change? Give an example where they change or prove they cannot change.