Assignment 9

CSCI 3110: Design and Analysis of Algorithms

Due July 23, 2019

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1. Consider the following dominance problem. Given a set S of points in 3-dimensional space, report for every point $p \in S$, all points $q \in S \setminus \{P\}$ such that $x(q) \geq x(p)$, $y(q) \geq y(p)$, and $z(q) \geq z(p)$. More precisely, the problem asks to find all pairs $(p, q) \in S \times S$ such that $x(q) \geq x(p)$, $y(q) \geq y(p)$, $z(q) \geq z(p)$, and $q \neq p$.

   (a) (5 pts) Consider the following algorithm to solve this problem using a 3-dimensional range tree: For each point $p \in S$, do a 3-dimensional range query to find the points that dominate p. How much space would this algorithm require and how long would this take to output t pairs?

   (b) (15 pts) Develop an algorithm that solves this problem in $O(n \log n + t)$ time, where t is the total number of reported pairs. (Hint: Use the sweep-line approach to reduce this problem to a set of two-dimensional problems and use the proper data structure to solve each two-dimensional problem.) As usual, give either a brief description of your approach along with pseudo-code or a detailed description that would be sufficient for someone to write the pseudocode.

   (c) (10 pts) Show that your algorithm is correct, that is it terminates (which should be trivial) and outputs every dominated pair and just the dominated pairs (induction and contradiction should both work well).

   (d) (10 pts) Show that the running time of your algorithm is indeed $O(n \log n + t)$.