Assignment 3

CSCI 3110: Design and Analysis of Algorithms

Due June 4, 2019

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Assignments are due on the due date before class and have to include this cover page. Plagiarism in assignment answers will not be tolerated. By submitting their answers to this assignment, the authors named above declare that its content is their original work and that they did not use any sources for its preparation other than the class notes, the textbook, and ones explicitly acknowledged in the answers. Any suspected act of plagiarism will be reported to the Faculty's Academic Integrity Officer and possibly to the Senate Discipline Committee. The penalty for academic dishonesty may range from failing the course to expulsion from the university, in accordance with Dalhousie University's regulations regarding academic integrity.

In many of the questions on this assignment you are asked to *describe* an algorithm. This means to explain in words how your algorithm would solve the problem and then give a brief justification that your method has the claimed running time and is correct. You may give pseudocode but it is not necessary.

1. Consider the following task.

   Input: A connected, undirected graph G.

   Question: Is there an edge you can remove from G while still leaving G connected?

   (a) (10 pts) Describe a linear-time ($\mathrm{O}(n+m)$) algorithm for solving this question.

   (b) (Bonus: 5 pts) Can you reduce the running time of your algorithm to $\mathrm{O}(n)$?

2. Consider the interval scheduling problem discussed in class. Suppose that instead of always selecting the interval that ends first, we instead select the interval that starts last and is compatible with all previously selected intervals. Explain how this approach is a greedy algorithm and prove that it yields an optimal solution by induction.

3. Consider an undirected graph $G = (V, E)$ with distinct nonnegative edge weights $w_e \geq 0$. Suppose that you have computed a minimum spanning tree of G, and that you have also computed shortest paths to all nodes from a particular node $s \in V$.

   Now suppose each edge weight is increased by 1: the new weights are $w'_e = w_e + 1$.

   (a) (10 pts) Does the minimum spanning tree change? Give an example where it changes or prove it cannot change.

   (b) (10 pts) Do the shortest paths change? Give an example where they change or prove they cannot change.

4. Here is a problem that occurs in automatic program analysis. For a set of variables $x_1, \ldots, x_n$, you are given some equality constraints, of the form $x_i = x_j$ and some disequality constraints, of the form $x_i \neq x_j$. Is it possible to satisfy all of them? For instance, the constraints $x_1 = x_2, x_2 = x_3, x_3 = x_4, x_1 \neq x_4$ cannot be satisfied.

   (a) (10 pts) Describe an $\mathrm{O}(m + n \lg n)$ time algorithm that takes as input m constraints over n variables and decides whether the constraints can be satisfied.

   (b) (Bonus: 5 pts) Can you reduce the running time of your algorithm to $\mathrm{O}(n+m)$?