Assignment 1

CSCI 3110: Design and Analysis of Algorithms

Due May 23, 2019

Banner ID: ____________ Name: ____________

Banner ID: ____________ Name: ____________

Banner ID: ____________ Name: ____________

Assignments are due on the due date before class and have to include this cover page. Plagiarism in assignment answers will not be tolerated. By submitting their answers to this assignment, the authors named above declare that its content is their original work and that they did not use any sources for its preparation other than the class notes, the textbook, and ones explicitly acknowledged in the answers. Any suspected act of plagiarism will be reported to the Faculty's Academic Integrity Officer and possibly to the Senate Discipline Committee. The penalty for academic dishonesty may range from failing the course to expulsion from the university, in accordance with Dalhousie University's regulations regarding academic integrity.

1. (10 pts) For each of the following pairs of functions, indicate whether $f = \mathrm{O}(g)$, $f = \Omega(g)$, or both (in which case $f = \Theta(g)$). You do not need to explain your answer.

	$f(n)$	$g(n)$
(a)	n	$3n$
(b)	$n-1$	$n+1$
(c)	$1000n$	$0.01n^2$
(d)	$10n^2$	n^3+n^2
(e)	n	$(\lg n)^2$
(f)	3^n	2^n
(g)	4^n	2^{2n}
(h)	n	$2^{\lg n}$
(i)	$\sum_{i=0}^{n} i$	n^2
(j)	$\sum_{i=0}^{n} \lg n$	$\sum_{i=0}^{\lg n} n$

2. (12 pts) Sort the following functions by increasing order of growth. For every pair of consecutive functions f(n) and g(n) in the sorted list, prove that $f(n) = \mathrm{o}(g(n))$.

$$\sqrt{n} \quad 4^n \quad 4^{\lg n} \quad \lg n$$

Hint: recall the limit rule for $\mathrm{o}(\cdot)$:

$$\lim_{n\to\infty} \frac{f(n)}{g(n)} = 0 \iff f(n) = \mathrm{o}(g(n))$$

3. (10 pts) For each of the following functions $f(n)$, prove the stated claim by providing constants n_0, c_1, and c_2 such that for all $n \geq n_0$, $c_1 g(n) \leq f(n)$ or $f(n) \leq c_2 g(n)$, and provide a calculation that shows that this inequality does indeed hold.

   (a) $f(n) = 2n^2 + 3n^3 - 50n\lg n + 10 = O(n^3) = O(g(n))$

   (b) $f(n) = 2n\log n + 3n^2 - 10n - 10 = \Omega(n^2) = \Omega(g(n))$