Connected graph $G$ - can you remove an edge that leaves it connected?

At least $n-1$ edges

b) A tree - $n$ vertices are there more than $n-1$ edges
$v_0 \rightarrow \text{all} \quad \text{Dijkstra's}\n$
\text{reverse the graph}\n\text{all} \rightarrow v_0
**Bonus Constraint Satisfaction**

Keep a graph, each constraint starts as a vertex

\[ x_1 = x_2 \quad x_2 = x_3 \quad x_3 = x_4 \quad x_1 \neq x_4 \]

1. Assign edges for each equality
2. Compute the connected components
3. Check all the disequality constraints to see if they are in the same component
Minimize waiting time

Customers: 3 5 7

Waiting: 3 8 15 16

Total: 42

Reworder: 1 3 5 7

Waiting: 1 4 9 16

Total: 30
Problem: Running Times

so we have a problem -

how long does it take to solve a problem?

ex: sorting \( O(n \log n) \) - merge sort

To show the \( O() \) running time of a problem you just need to give one algorithm that solves it in that time.
How to prove a lower bound on a problem?

Can't just analyze one algorithm.
Need to analyze all possible algorithms!
Sorting

assumption: comparison-based

is \( A_i \leq A_{i-j} \)

represent any sorting algorithm as a tree.

proof on Page 63
Sort a 3-element array

\[ a_1, a_2, a_3 \]

9 3 5 \(\Rightarrow\) 3 5 9
Sort a 3-element array

$a_1 < a_2$
- No
- $a_2 < a_3$?
  - No
  - $a_3 < a_3$?
    - No
    - $a_3 < a_3$?
      - No
      - $a_2 < a_3$
        - $a_3 < a_3$
          - No
          - No
        - $a_2 < a_3$
          - Yes
          - $a_1 3$
  - Yes
    - $a_2 < a_3$
      - Yes
      - $a_1 3$

3 2 1
2 3 1

we don't have the permutation 312!

$9 \ 3 \ 5 \Rightarrow 3 \ 5 \ 9$

$4 \ 6 \ 2 \Rightarrow 4 \ 2 \ 6$

$a_1, a_2, a_3, a_1$

$a_1, a_2, a_3$

$a_1, a_3, a_2$
every permutation must be a leaf

$n$ elements $\to n!$ permutations $\geq n$ leaves

height? $\geq \log(n!/c^n)$

$n! \geq \left(\frac{n}{2}\right)^{\frac{n}{2}}$

$\geq \frac{n}{c} \log \frac{n}{c}$

- You need $\Omega(n \log n)$ comparisons to sort

$\Theta(n \log n)$
How to merge $k$ arrays of size $n$

$$\underbrace{n \quad n \quad n \quad \cdots \quad n}_{k}$$
7 3 9 5 1

\[ m_{\text{in}} = 1 \]

\[ \max = 9 \]

\[ m = 8 \]

Can you sort in \( O(n + M) \) time?