Relation between Topological Organization and Learning Ability of Neural Networks

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Abstract The objective of this study is to investigate the correlation between the internal topological organization in neural network and the learning ability of the neural network. This study is motivated by the interesting neurophysiological examination that shows the significance of topographic map of adult mammals’ brains to their learning ability and plasticity. In this study we propose a model of a layered neural network with Self-Organizing Map in its hidden layer which is connected to Perceptron as a learning part. We run several simulations to show the significant of the topological order in helping the learning process and relearning process.

Key words Self-Organizing map (SOM), Perceptron, Topological Representation, Cortical Plasticity, Unsupervised Learning, Supervised Learning

1. Introduction

Self-Organizing map has been a principle model for explaining experience-driven development and learning in the brain ([1], [2], [3], [4], [5], and [6]). This paper provides initial experiments on the relation between the topological representations the external stimuli and the learning ability of a learning system.

This work is partly motivated by the existence of a biological experiment in investigating the significance of the topographic map of adult mammals’ brain for learning ([7], [8], and [9]).

For investigating the correlation between topological order and the learning ability of a leaning system, we modified a layered neural network model called Map Initialized Perceptron (MIP) ([11] and [12]). This layered model includes a topological map in its hidden layer to internally represent and organize the external stimuli, and a feed forward layer (Perceptron layer). We are aware that there are several models that inherently combined self-organization and learning as in [13] and [14]. However, learning in these models is inherently included in the self-organization process, hence it will be difficult to utilize this model for our study, which requires the investigation on the relation between the fidelity
of topological organization with the learning ability.

This paper is organized as follows. Section 2 explains the dynamics and learning process of MIP. In Section 3 we run experiments to investigate the significance of topological organization to the learning ability of the model. We also argued about the importance of the topological fidelity by gradually distant the representation of the topological layer through the deletion of its hidden neurons. Here, plastically and the relearning ability of the model is also explained. Final Section is for discussion and future works.

2. Map- Initialized Perceptron

For the purpose of investigating the correlation between the fidelity of topological representation and the learning ability of neural networks, we utilize a three-layered neural network model called Map initialized Perceptron [10], [11]. Similar to MLP [15], MIP consists of input, hidden and output layers. The input layer receives external sensory inputs, which are then topological mapped into the hidden layer. The neurons in the hidden layer generate outputs which reflects the topological characteristics inherent in that layer and forward the information to be processed by the output layer.

The structure of MIP is illustrated in Fig. 2, where the hidden layer is one dimensional topographic map.

The training process of MIP consists of two stages, unsupervised training for the formation of topographical map in the hidden layer and the supervised training for generating external outputs. There is no limitation on the training problems, but for clarity and simplicity, in this report we provide the model with a basic task of coding analog signals observed in the input layer, into their digital representations in the output layer.

2.1 Dynamics of MIP

Here the task of MIP is to decode continuous value between 0 and 2π given as input into their digital representation in the output layer. For this task we set the number of hidden neurons to 150 and the output neurons to 100, respectively. The hidden neurons are evenly intervals in one dimensional grid to form a one dimensional topographic map. To avoid distortions in topological representations of the edges of the input signal, we use a cyclic (periodic) map in the hidden layer, where the distance between the first hidden neuron and the last one is assumed to be 1.

When MIP observes an input \( r^n(t) \), it measures the distance between the input and the reference vector \( c_i(t) \) encoded by the \( i \)-th hidden neuron, \( d_i(t) \) as follows.

The winner is chosen followed Eq. 2. \( R \) is the max value of input range.

\[
d_i(t) = \min(|c_i(t) - r^n(t)|, R - |c_i(t) - r^n(t)|) \quad (1)
\]

\[
\text{win} = \arg \max_\lambda (\exp(-d_i(t)^2)) \quad (2)
\]

The reference \( c_i \) is then modified according to Eq. 3.

\[
c_i(t+1) = c_i(t) + \alpha_{som} \Lambda(d_{win}(t)) \cdot d_{f_i}(t) \quad (3)
\]

Where \( d_{f_i} \) is the modification value defined in Eq. 4.

\[
d_{f_i}(t) = \left\{ \begin{array}{ll}
-(t - (r^n(t) - c_i(t))) & (r^n(t) > c_i(t) \wedge r^n(t) - c_i(t) > \frac{1}{2}) \\
(+t + (r^n(t) - c_i(t))) & (r^n(t) < c_i(t) \wedge c_i(t) - r^n(t) > \frac{1}{2})
\end{array} \right. \quad (4)
\]

After the formation of organization map, the second stage of the learning process is executed. In this stage, the connections between the hidden layer and the output layers are trained in a supervised manner.

The value of the \( j \)-th output at time \( t \), \( o_j \) is calculated according to Eq. 5. Here, \( R \) is the maximum value of the input.

\[
o_j(t) = \sum_{i=1}^{N} (r_i^{out}(t) \cdot w_{ij}(t)) \quad (5)
\]
\( r_i^{\text{out}}(t) = \exp\left(\frac{-d_i(t)^2}{2(\eta)^2}\right) \)  \hfill (6)

The object of the learning is to minimize the error, \( E \), defined as follows.

\[ E(t) = \sum_j (o_j^{\text{out}}(t) - T_j(t))^2 \]  \hfill (7)

\( T_j \) is the teacher signal for the \( j \)-th output neuron defined in Eq. 8 and Eq. 9.

\[ d_j(t) = \min\left( |j \cdot \frac{R}{M} - r^{in}(t)|, |j \cdot \frac{R}{M} - r^{in}(t)| \right) \]  \hfill (8)

\[ T_j(t) = \exp\left(\frac{-d_j(t)^2}{2(\eta)^2}\right) \]  \hfill (9)

The modification of the connection weights is executed according to Eq. 10.

\[ w_{ij}(t+1) = w_{ij}(t) + \alpha \cdot (T_j(t) - o_j(t)) \cdot r_i^{\text{out}}(t) \]  \hfill (10)

Here, \( \eta \) is the learning rate which is empirically set as 0.1.

3. Experiments

3.1 Effect of topological order to the learning ability

For analyzing the effect of the topological order to the learning ability, we run three experiments. In the first one, we executed the Perceptron learning in MIP without organizing a topological map in the hidden layer. In this case, the external input are not topological represented in the hidden layer. In the second experiment, the topographic map was to some extent trained prior to the execution of the Perceptron learning, while in the final case, the topographic map was exhaustively trained prior to the Perceptron learning.

Fig. 3 shows the learning curve of Perceptron training. We can clearly observe that the Perceptron layer can be rapidly trained when the fidelity of the topological organization is high. We can learn that topological order in the hidden layer significantly helps the training process.

3.2 Reorganization and Retraining

In this experiment we analyzed the fidelity of MIP, when the topological order in the hidden layered is disturbed. We simulate the disturbance by randomly removing a certain number of hidden neurons. This has a coarse biological analogy with a partially damaged brain. The objective is to observe how the reorganization and relearning can help to recover the function of the learning system.

In these experiments, we removed 5-95% of hidden neurons in a consecutive manner, and executes two types of relearning. In the first case, the map in the hidden layer is reorganized to capture alternative topological order using the rest of the neurons, prior to the retraining of Perceptron. The first case is denoted as “SOM and Perceptron” in Fig. 4. In the second case, Perceptron is retrained without reorganizing the topographic map in the hidden layer.

The experiments results are given in Fig. 4, where we can observe that when the percentage of the damaged neurons exceeds 15%, the training quality of the Perceptron gradually deteriorates if the topographic map is not reorganized. This result is understandable, in that, the topographic map does not have sufficient ability to represent the underlying topological characteristics of the external inputs. We can also observe that reorganization in the hidden layer helps to support the retraining process until the number of the damaged neurons reaches a level where the hidden layer lost its ability to topologically represent the input. "Error of the Center of Mass" in Fig. 4 refers to the diffraction between the output neurons and their teacher signals, talking the cyclic characteristics into account.

These experiments shows that topological order is crucial in supporting not only the supervised learning process but also the relearning process.

![Error of Center of Mass](image)

**图 4 Disturbance in Topological Representation**

Fig. 5 shows the learning curve of Perceptron with regards to various percentages of the damaged neurons. These graphs show that the neural network can sustain its functionality although it lost 80% of the hidden neurons as long as
the topographic map in the hidden layer is reorganized using the rest of the neurons. It is also clear that relearning of Perceptron without reorganization fails when the percentage of the damaged neurons exceeds 50%.

![Image 1](image1)

**Fig. 5** Topological Disturbance and Learning Ability

20% neurons were deleted

50% neurons were deleted

80% neurons were deleted

95% neurons were deleted

<table>
<thead>
<tr>
<th>error</th>
<th>learning epoch</th>
</tr>
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<tbody>
<tr>
<td>0</td>
<td>10 20 30 40 50</td>
</tr>
</tbody>
</table>

![Image 2](image2)

**Fig. 6** Topological Disturbance and External Output

20% neurons were deleted

50% neurons were deleted

80% neurons were deleted

95% neurons were deleted

<table>
<thead>
<tr>
<th>error</th>
<th>input range</th>
</tr>
</thead>
<tbody>
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<td>0</td>
<td>$\pi 2\pi$</td>
</tr>
</tbody>
</table>

![Image 3](image3)

**Fig. 7** Lost and Hidden Neurons and Topological Representation

4. Conclusion and Future Work

In this study, utilizing a modified MIP we run several simulations for arguing the importance of topological representation and the learning ability of a learning system. Our experiments showed that the quality of the topological representation helps the neural network in forwarding the supervised learning process. The results also show the plasticity of the topological representation and the learning system, where relearning of a partially damaged representation is possible as long as the internal topological organization is not excessively damaged.

Our immediate future works is to implement a physiological topological map model in the middle layer of MIP. We plan to utilize Dynamic Neural Field (DNF) [2], which, unlike conventional SOM which only works with one input, can work with multiple inputs. We are interested in investigating the quality of the self-organization in the existence of top-down attention and also noise, and its relation with leaning ability.

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