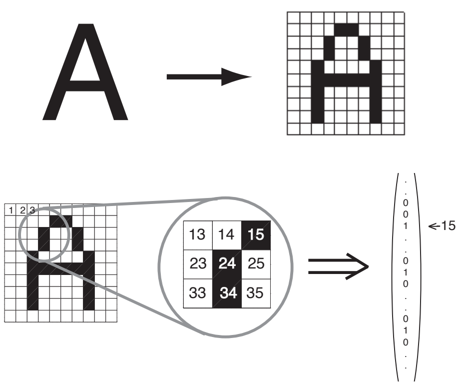
Fundamentals of Computational Neuroscience 2e

December 27, 2009

Chapter 6: Feed-forward mapping networks

Digital representation of a letter



Optical character recognition: Predict meaning from features.
E.g., given features **x**, what is the character **y**

$$f : \mathbf{x} \in \mathbf{S}_1^n \rightarrow \mathbf{y} \in \mathbf{S}_2^m$$

Examples given by lookup table

Boolean AND function

x_1	x_2	y
0	0	1
0	1	0
1	0	0
1	1	1

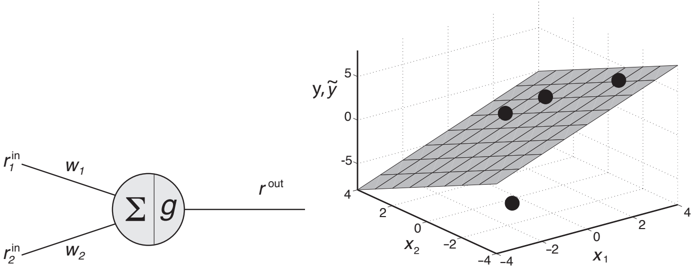
Look-up table for a non-boolean example function

x_1	x_2	y
1	2	-1
2	1	1
3	-2	5
-1	-1	7
...	...	...

The population node as perceptron

Update rule: $\mathbf{r}^{\text{out}} = g(\mathbf{w}\mathbf{r}^{\text{in}})$ (component-wise: $r_i^{\text{out}} = g(\sum_j w_{ij} r_j^{\text{in}})$)
For example: $r_i^{\text{in}} = x_i$, $\tilde{y} = r^{\text{out}}$, linear grain function $g(x) = x$:

$$\tilde{y} = w_1 x_1 + w_2 x_2$$



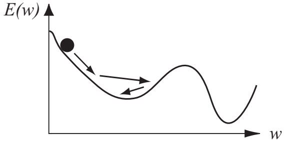
How to find the right weight values?

Objective (error) function, for example: mean square error (MSE)

$$E = \frac{1}{2} \sum_i (r_i^{\mathrm{out}} - y_i)^2$$

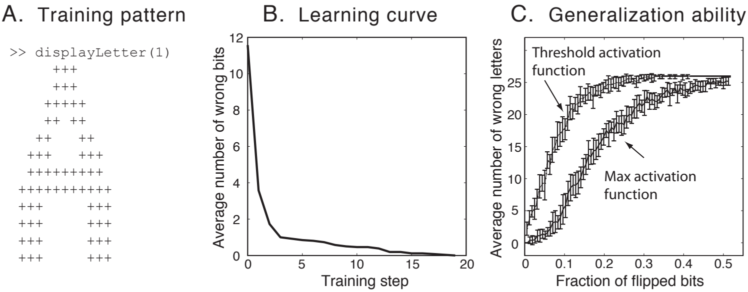
Gradient descent method: $w_{ij} \leftarrow w_{ij} - \epsilon \frac{\partial E}{\partial w_{ij}}$

$$= w_{ij} - \epsilon (y_i - r_i^{\mathrm{out}}) r_j^{\mathrm{in}}$$ for MSE, linear gain



Initialize weights arbitrarily
Repeat until error is sufficiently small
    Apply a sample pattern to the input nodes: $r_i^0 = r_i^{\mathrm{in}} = \xi_i^{\mathrm{in}}$
    Calculate rate of the output nodes: $r_i^{\mathrm{out}} = g(\sum_j w_{ij} r_j^{\mathrm{in}})$
    Compute the delta term for the output layer: $\delta_i = g'(h_i^{\mathrm{out}})(\xi_i^{\mathrm{out}} - r_i^{\mathrm{out}})$
    Update the weight matrix by adding the term: $\Delta w_{ij} = \epsilon \delta_i r_j^{\mathrm{in}}$

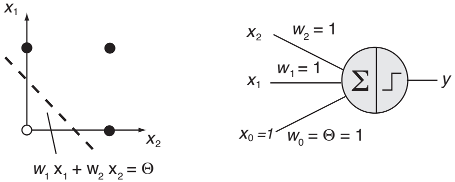
Example: OCR


A. Training pattern
>> displayLetter(1)
B. Learning curve
Average number of wrong bits
Training step
C. Generalization ability
Threshold activation function
Max activation function
Average number of wrong letters
Fraction of flipped bits

Example: Boolean function

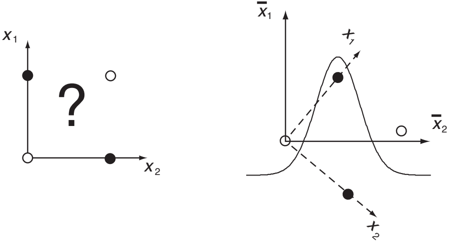
A. Boolean OR function

x_1	x_2	y
0	0	0
0	1	1
1	0	1
1	1	1



B. Boolean XOR function

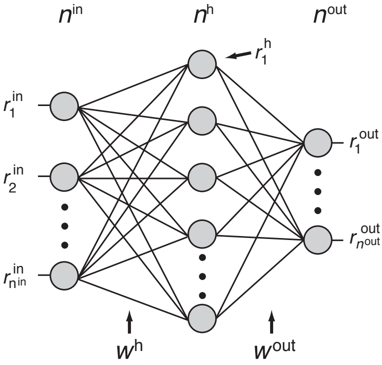
x_1	x_2	y
0	0	0
0	1	1
1	0	1
1	1	0



perceptronTrain.m

```
%% Letter recognition with threshold perceptron
 clear; clf;
 nIn=12*13; nOut=26;
 wOut=rand(nOut,nIn)-0.5;

% training vectors
 load pattern1;
 rIn=reshape(pattern1’, nIn, 26);
 rDes=diag(ones(1,26));

% Updating and training network
 for training_step=1:20;
     % test all pattern
      rOut=(wOut*rIn)>0.5;
      distH=sum(sum((rDes-rOut).^2))/26;
      error(training_step)=distH;
     % training with delta rule
      wOut=wOut+0.1*(rDes-rOut)*rIn’;
 end

 plot(0:19,error)
 xlabel(’Training step’)
 ylabel(’Average Hamming distance’)
```

The mulitlayer Perceptron (MLP)



Update rule: $\mathbf{r}^{\text{out}} = g^{\text{out}}(\mathbf{w}^{\text{out}} g^{\text{h}}(\mathbf{w}^{\text{h}} \mathbf{r}^{\text{in}}))$

Learning rule (error backpropagation): $w_{ij} \leftarrow w_{ij} - \epsilon \frac{\partial E}{\partial w_{ij}}$

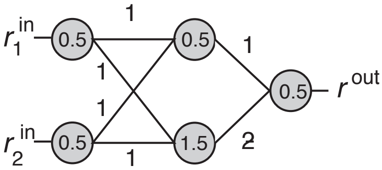
The error-backpropagation algorithm

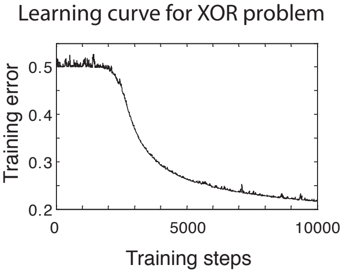
Initialize weights arbitrarily
Repeat until error is sufficiently small
- Apply a sample pattern to the input nodes: $r_i^0 := r_i^{\mathrm{in}} = \xi_i^{\mathrm{in}}$
- Propagate input through the network by calculating the rates of nodes in successive layers l: $r_i^l = g(h_i^l) = g(\sum_j w_{ij}^l r_j^{l-1})$
- Compute the delta term for the output layer: $\delta_i^{\mathrm{out}} = g'(h_i^{\mathrm{out}})(\xi_i^{\mathrm{out}} - r_i^{\mathrm{out}})$
- Back-propagate delta terms through the network: $\delta_i^{l-1} = g'(h_i^{l-1}) \sum_j w_{ji}^l \delta_j^l$
- Update weight matrix by adding the term: $\Delta w_{ij}^l = \epsilon \delta_i^l r_j^{l-1}$

mlp.m

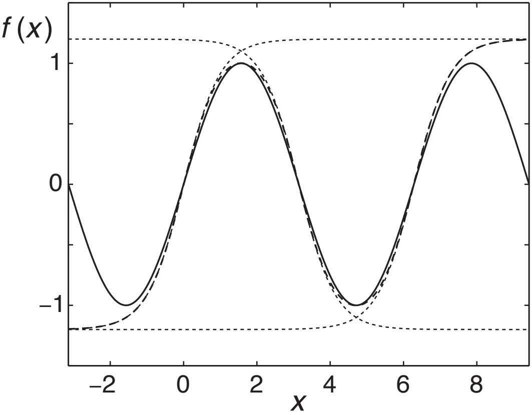
```
%% MLP with backpropagation learning on XOR problem
 clear; clf;
 N_i=2; N_h=2; N_o=1;
 w_h=rand(N_h,N_i)-0.5; w_o=rand(N_o,N_h)-0.5;

 % training vectors (XOR)
 r_i=[0 1 0 1 ; 0 0 1 1];
 r_d=[0 1 1 0];

 % Updating and training network with sigmoid activation function
 for sweep=1:10000;
   % training randomly on one pattern
     i=ceil(4*rand);
     r_h=1./(1+exp(-w_h*r_i(:,i)));
     r_o=1./(1+exp(-w_o*r_h));
     d_o=(r_o.*(1-r_o)).*(r_d(:,i)-r_o);
     d_h=(r_h.*(1-r_h)).*(w_o'*d_o);
     w_o=w_o+0.7*(r_h*d_o')';
     w_h=w_h+0.7*(r_i(:,i)*d_h')';
   % test all pattern
     r_o_test=1./(1+exp(-w_o*(1./(1+exp(-w_h*r_i)))));
     d(sweep)=0.5*sum((r_o_test-r_d).^2);
 end
 plot(d)
```

MLP for XOR function

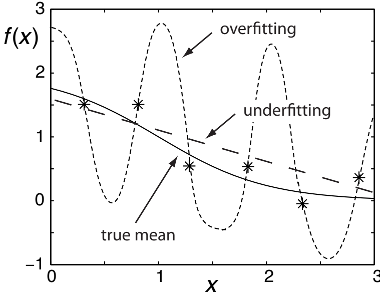




MLP approximating sine function



Overfitting and underfitting

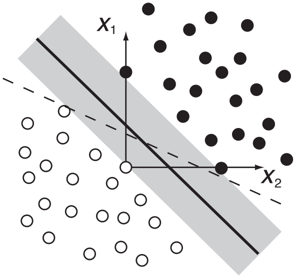


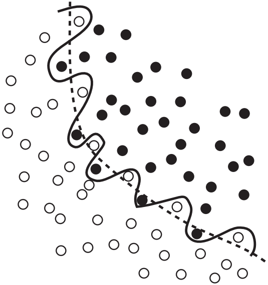
Regularization, for example

$$E = \frac{1}{2}\sum_i (r_i^{\mathrm{out}} - y_i)^2 - \gamma_r \frac{1}{2}\sum_i w_i^2$$

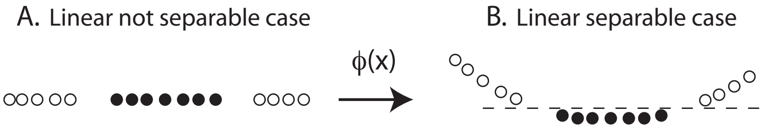
Support Vector Machines

Linear large-margine classifier





SVM: Kernel trick



Further Readings

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