Fundamentals of Computational Neuroscience 2e

December 28, 2009

Chapter 4: Associators and synaptic plasticity

Types of plasticity

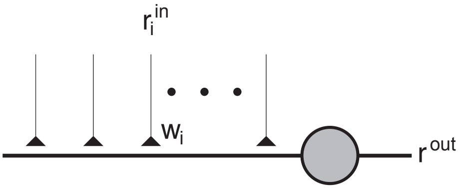
- **Structural plasticity** is the mechanism describing the generation of new connections and thereby redefining the topology of the network.
- **Functional plasticity** is the mechanism of changing the strength values of existing connections.

Hebbian plasticity

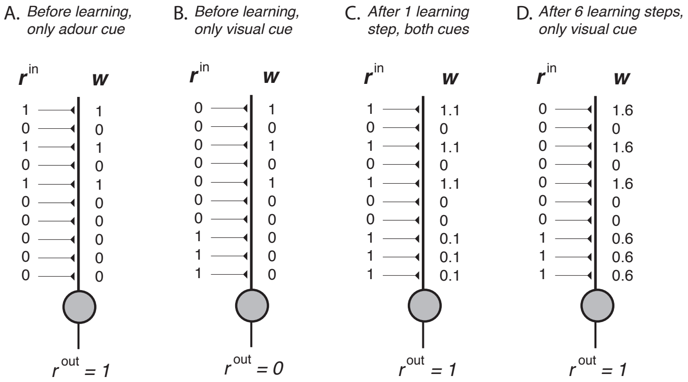
”When an axon of a cell A is near enough to excite cell B or repeatedly or persistently takes part in firing it, some growth or metabolic change takes place in both cells such that A’s efficiency, as one of the cells firing B, is increased.”

Donald O. Hebb, **The organization of behavior**, 1949
See also Sigmund Freud, **Law of association by simultaneity**, 1888

Association



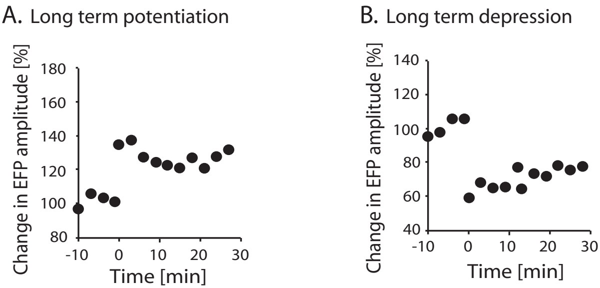
Neuron model: In each time step the model neurons fires if $\sum_i w_i r_i^{\text{in}} > 1.5$
Learning rule: Increase the strength of the synapses by a value $\Delta w = 0.1$ if a presynaptic firing is paired with a postsynaptic firing.


A. Before learning, only adour cue
B. Before learning, only visual cue
C. After 1 learning step, both cues
D. After 6 learning steps, only visual cue
r in
w
1 0 1 0 1 0 0 0 0 0
1 0 1 0 1 0 0 0 0 0
r out = 1
0 0 0 0 0 0 0 1 1 1
1 0 1 0 1 0 0 0 0 0
r out = 0
1 0 1 0 1 0 0 1 1 1
1.1 0 1.1 0 1.1 0 0 0.1 0.1 0.1
r out = 1
0 0 0 0 0 0 0 1 1 1
1.6 0 1.6 0 1.6 0 0 0.6 0.6 0.6
r out = 1

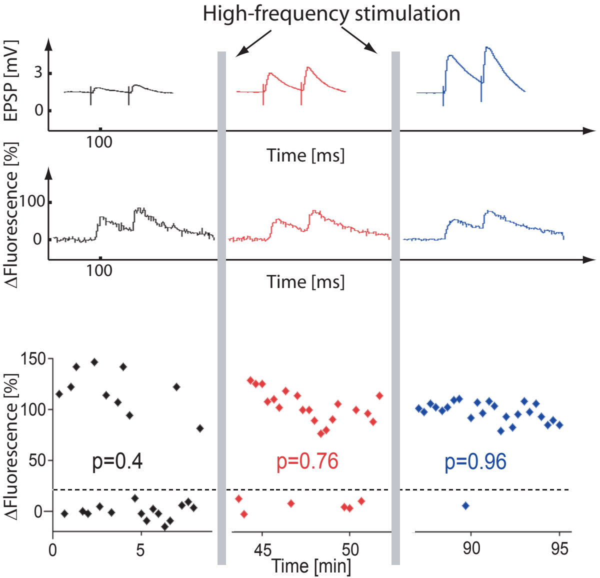
Features of associators and Hebbian learning

- Pattern completion and generalization
- Prototypes and extraction of central tendencies
- Graceful degradation and fault tolerance

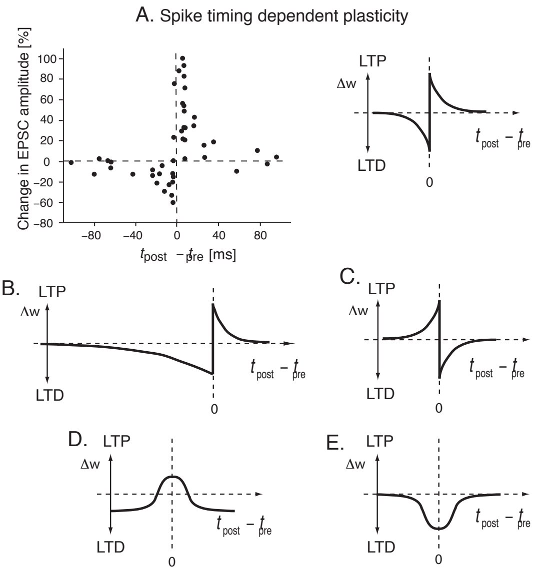
Classical LTP and LTD



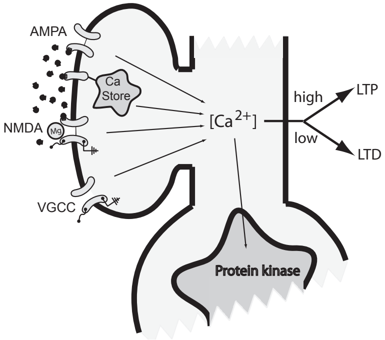
Synaptic neurotransmitter release probability

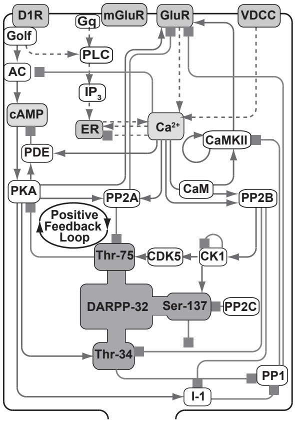


Spike timing dependent plasticity



The calcium hypothesis and modelling chemical pathways





Mathematical formulation of Hebbian plasticity

$$w_{ij}(t+\Delta t) = w_{ij}(t) + \Delta w_{ij}(t_i^{\mathrm{f}}, t_j^{\mathrm{f}}, \Delta t; w_{ij}).$$

$$\Delta w_{ij}^{\pm} = \epsilon^{\pm}(w) \mathrm{e}^{\mp \frac{t^{\mathrm{post}} - t^{\mathrm{pre}}}{\tau^{\pm}}} \; \Theta(\pm[t^{\mathrm{post}} - t^{\mathrm{pre}}]).$$

Additive rule with hard (absorbing) boundaries:

$$\epsilon^{\pm} = \begin{cases} a^{\pm} & \text{for } \; w_{ij}^{\mathrm{min}} \leq w_{ij} \leq w_{ij}^{\mathrm{max}} \\ 0 & \text{otherwise} \end{cases} ,$$

Multiplicative rule (soft boundaries):

$$\begin{aligned} \epsilon^{+} &= a^{+}(w^{\mathrm{max}} - w_{ij}) \\ \epsilon^{-} &= a^{-}(w_{ij} - w^{\mathrm{min}}). \end{aligned} \tag{1}$$

Hebbian learning in population and rate models

General: $\Delta w_{ij} = \epsilon(t, w)[f_{\text{post}}(r_i) f_{\text{pre}}(r_j) - f(r_i, r_j, w)]$

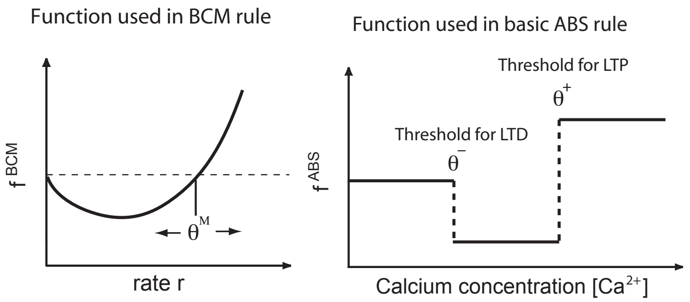
Mnemonic equation (Caianiello): $\Delta w_{ij} = \epsilon(w)[r_i r_j - f(w)]$

Basic Hebb: $\Delta w_{ij} = \epsilon r_i r_j$

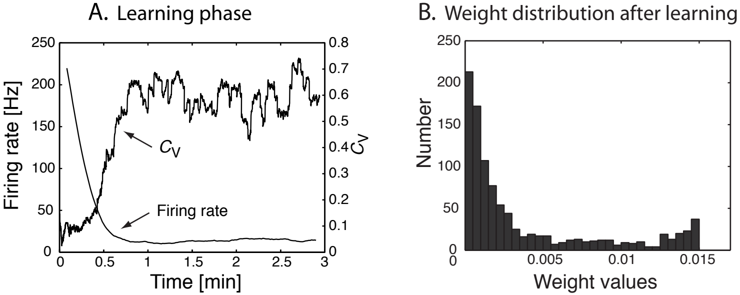
Covariance rule: $\Delta w_{ij} = \epsilon(r_i - \langle r_i \rangle)(r_j - \langle r_j \rangle)$

BCM theory: $\Delta w_{ij} = \epsilon(f^{\text{BCM}}(r_i; \theta^M)(r_j) - f(w))$

ABS rule: $\Delta w_{ij} = \epsilon(f_{\text{ABS}}(r_i; \theta^-, \theta^+)\text{sign}(r_j - \theta^{\text{pre}}))$



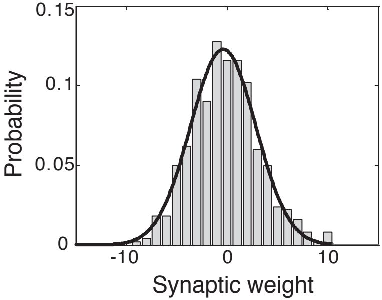
Synaptic scaling and weight distributions



after Song, Miller and Abbott 2000

Hebbian rate rules on random pattern

$$w_{ij} = \frac{1}{\sqrt{N_p}} \sum_{\mu} (r_i^{\mu} - \langle r_i \rangle)(r_j^{\mu} - \langle r_j \rangle).$$



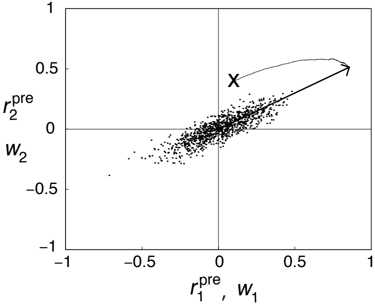
Synaptic scaling and PCA

Explicit normalization: $w_{ij} \leftarrow \frac{w_{ij}}{\sum_j w_{ij}}$

Basic decay: $\Delta w_{ij} = r_i r_j - c w_{ij}$

Willshaw rule: $\Delta w_{ij} = (r_i - w_{ij}) r_j$

Oja rule: $\Delta w_{ij} = r_i r_j - (r_i)^2 w_{ij}$



Further Readings

Laurence F. Abbott and Sacha B. Nelson (2000), **Synaptic plasticity: taming the beast**, in **Nature Neurosci. (suppl.)**, 3: 1178–83.

Alain Artola and Wolf Singer (1993), **Long-term depression of excitatory synaptic transmission and its relationship to long-term potentiation**, in **Trends in Neuroscience** 16: 480–487.

Mark C. W. van Rossum, Guo-chiang Bi, and Gina G. Turrigiano (2000) **Stable Hebbian learning from spike timing-dependent plasticity**, in **J. Neuroscience** 20(23): 8812–21