Fundamentals of Computational Neuroscience 2e

December 28, 2009

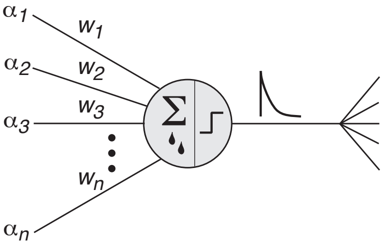
Chapter 3: Simplified neuron and population models

The leaky integrate-and-fire neuron

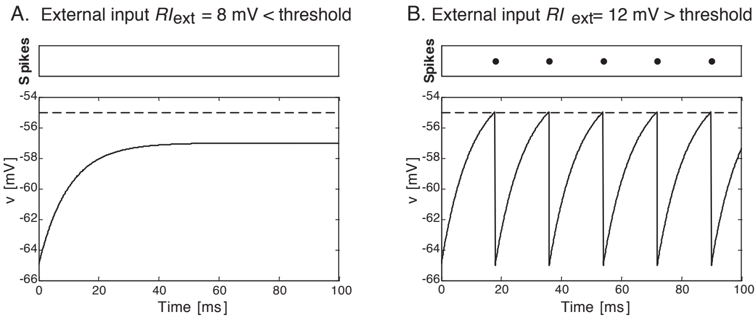
$$\tau_{\mathrm{m}} \frac{\mathrm{d}v(t)}{\mathrm{d}t} = -(v(t) - E_L) + RI(t), \tag{1}$$

$$v(t^{\mathrm{f}}) = \vartheta. \tag{2}$$

$$\lim_{\delta \to 0} v(t^{\mathrm{f}} + \delta) = v_{\mathrm{res}}, \tag{3}$$



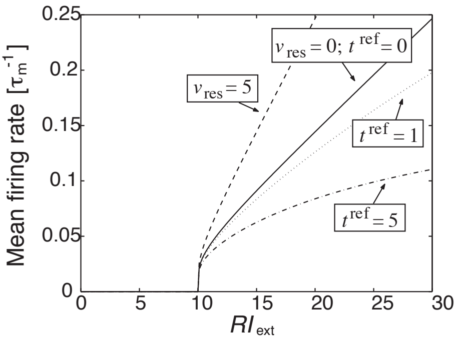
The leaky integrate-and-fire neuron (cont.)



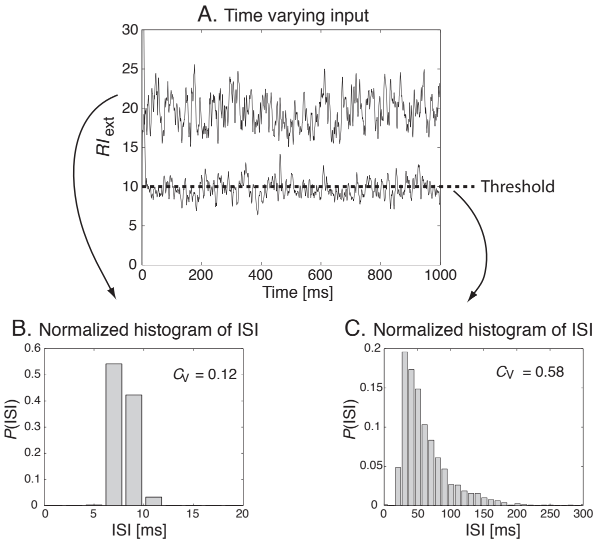
The LIF-neuron (cont.): Gain function

The inverse of the first passage time defines the **firing rate**

$$\bar{r} = (t^{\mathrm{ref}} - \tau_{\mathrm{m}} \ln \frac{\vartheta - RI}{v_{\mathrm{res}} - RI})^{-1} \tag{4}$$



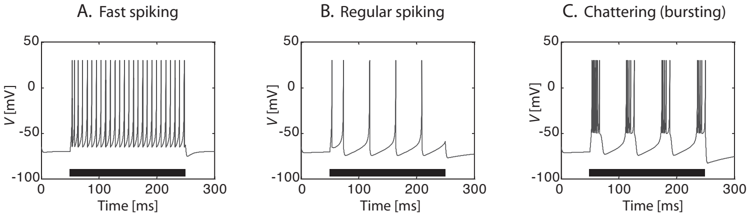
The LIF-neuron (cont.): Noise



The Izhikevich neuron

$$\frac{\mathrm{d}v(t)}{\mathrm{d}t} = 0.04v^2 + 5v + 140 - u + I(t)$$
$$\frac{\mathrm{d}u(t)}{\mathrm{d}t} = a(bv - u)$$

$$v(v > 30) = c \;\; \text{and} \;\; u(v > 30) = u - d$$



McCulloch-Pitts neuron

$$h = \sum_i x_i^{\text{in}}$$

$$x^{\text{out}} = \begin{cases} 1 & \text{if } h > \Theta \\ 0 & \text{otherwise} \end{cases}$$

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VOLUME 5, 1943

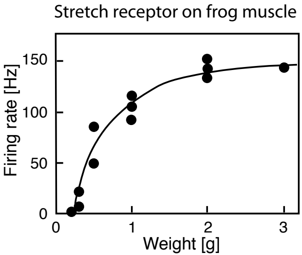
A LOGICAL CALCULUS OF THE
IDEAS IMMANENT IN NERVOUS ACTIVITY

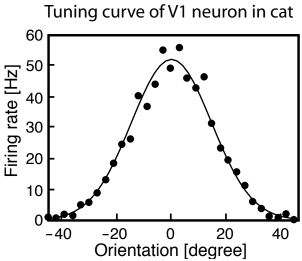
WARREN S. MCCULLOCH AND WALTER PITTS

FROM THE UNIVERSITY OF ILLINOIS, COLLEGE OF MEDICINE,
DEPARTMENT OF PSYCHIATRY AT THE ILLINOIS NEUROPSYCHIATRIC INS
AND THE UNIVERSITY OF CHICAGO

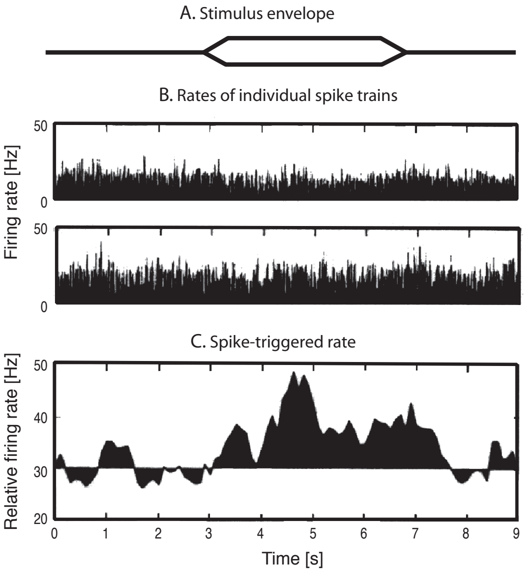
Because of the "all-or-none" character of nervous activity, r
events and the relations among them can be treated by means of p
sitional logic. It is found that the behavior of every net can be desc
in these terms, with the addition of more complicated logical mear
nets containing circles; and that for any logical expression satis

The firing rate hypothesis



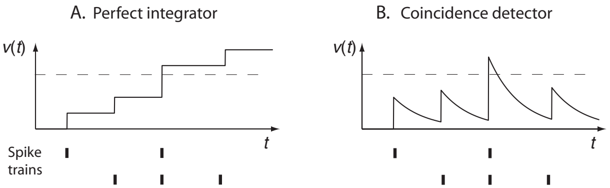


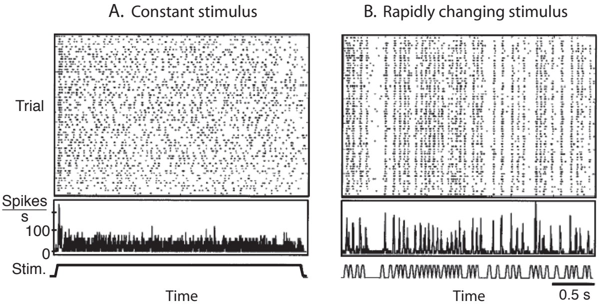
Counter example: correlation code (?)



From DeCharms and Merzenich 1996

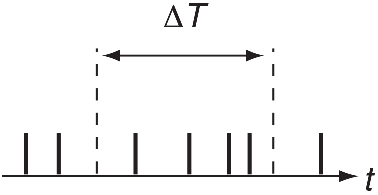
Integrator or coincidence detector?

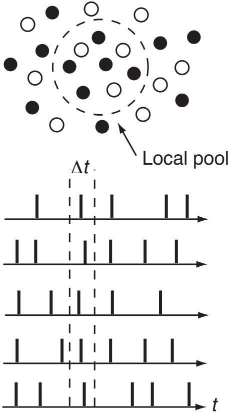




From Buracas et al. 1998

Population model





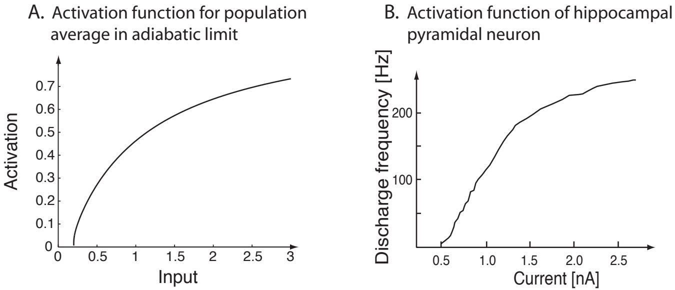
Population dynamics

For slow varying input (adiabatic limit), when all nodes do practically the same, same input, etc (Wilson and Cowan, 1972):

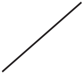
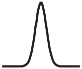
$$\tau \frac{\mathrm{d}A(t)}{\mathrm{d}t} = -A(t) + g(RI^{\mathrm{ext}}(t)). \tag{5}$$

Gain function:

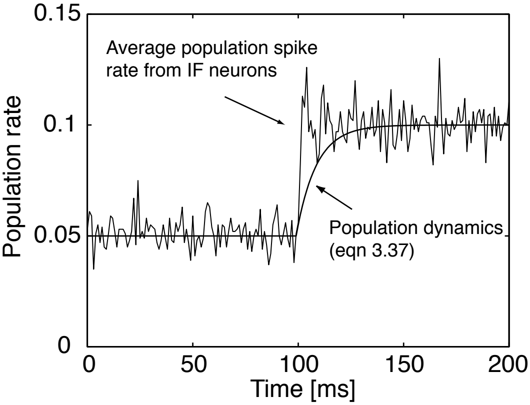
$$g(x) = \frac{1}{t^{\mathrm{ref}} - \tau \log(1 - \frac{1}{\tau x})}, \tag{6}$$



Other gain functions

Type of function	Graphical represent.	Mathematical formula	MATLAB implementation
Linear		$g^{\text{lin}}(x) = x$	`X`
Step		$g^{\text{step}}(x) = \begin{cases} 1 & \text{if } x > 0 \\ 0 & \text{elsewhere} \end{cases}$	`floor(0.5*(1+sign(x)))`
Threshold-linear		$g^{\text{theta}}(x) = x\,\Theta(x)$	`x.*floor(0.5*(1+sign(x)))`
Sigmoid		$g^{\text{sig}}(x) = \frac{1}{1+\exp(-x)}$	`1./(1+exp(-x))`
Radial-basis		$g^{\text{gauss}}(x) = \exp(-x^2)$	`exp(-x.^2)`

Fast population response!!!



Further Readings

Wolfgang Maass and Christopher M. Bishop (eds.) (1999), **Pulsed neural networks**, MIT Press.

Wulfram Gerstner (2000), **Population dynamics of spiking neurons: fast transients, asynchronous states, and locking**, in **Neural Computation** 12: 43–89.

Eugene M. Izhikevich (2003), **Simple Model of Spiking Neurons**, in **IEEE Transactions on Neural Networks**, 14: 1569–1072.

Eugene M. Izhikevich (2004), **Which model to use for cortical spiking neurons?**, in **IEEE Transactions on Neural Networks**, 15: 1063–1070.

Warren McCulloch and Walter Pitts (1943) **A logical calculus of the ideas immanent in nervous activity**, in**Bulletin of Mathematical Biophysics** 7:115–133.

Huge R. Wilson and Jack D. Cowan (1972), **Excitatory and inhibitory interactions in localized populations of model neurons**, in **Biophys. J.** 12:1–24.

Nicolas Brunel and Xiao-Jing Wang, (2001), **Effects of neuromodulation in a cortical network model of working memory dominated by recurrent inhibition**, in **Journal of Computational Neuroscience** 11: 63–85.