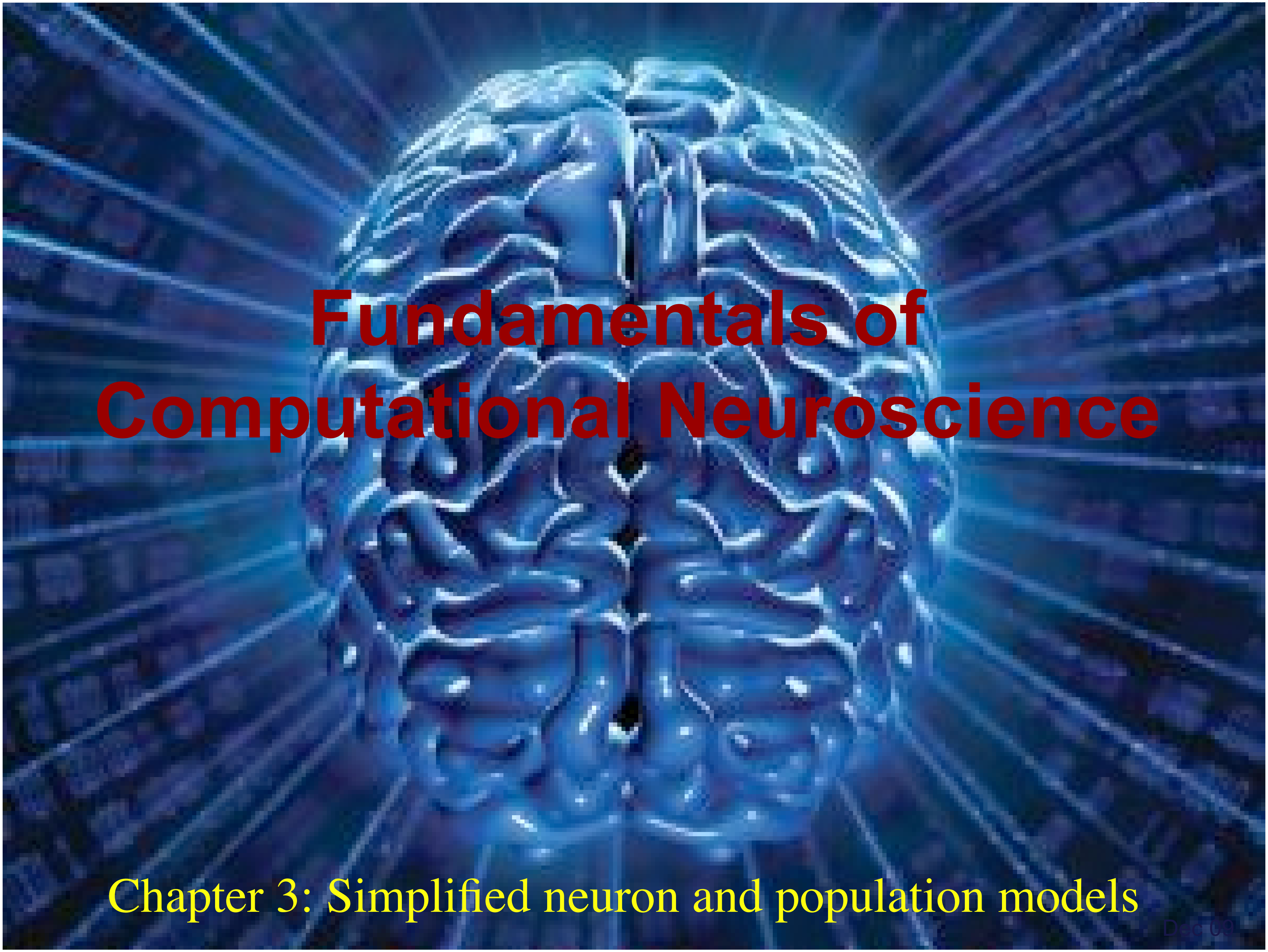


Fundamentals of Computational Neuroscience

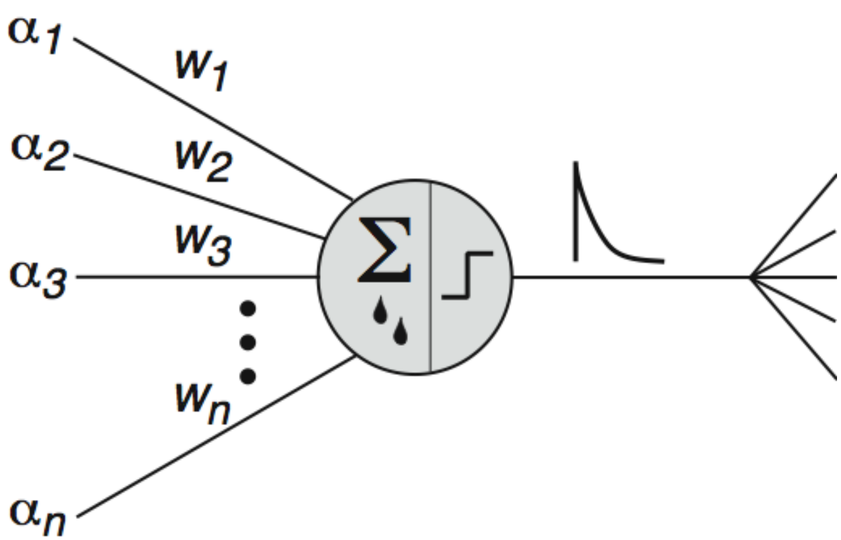
Chapter 3: Simplified neuron and population models

The leaky integrate-and-fire neuron

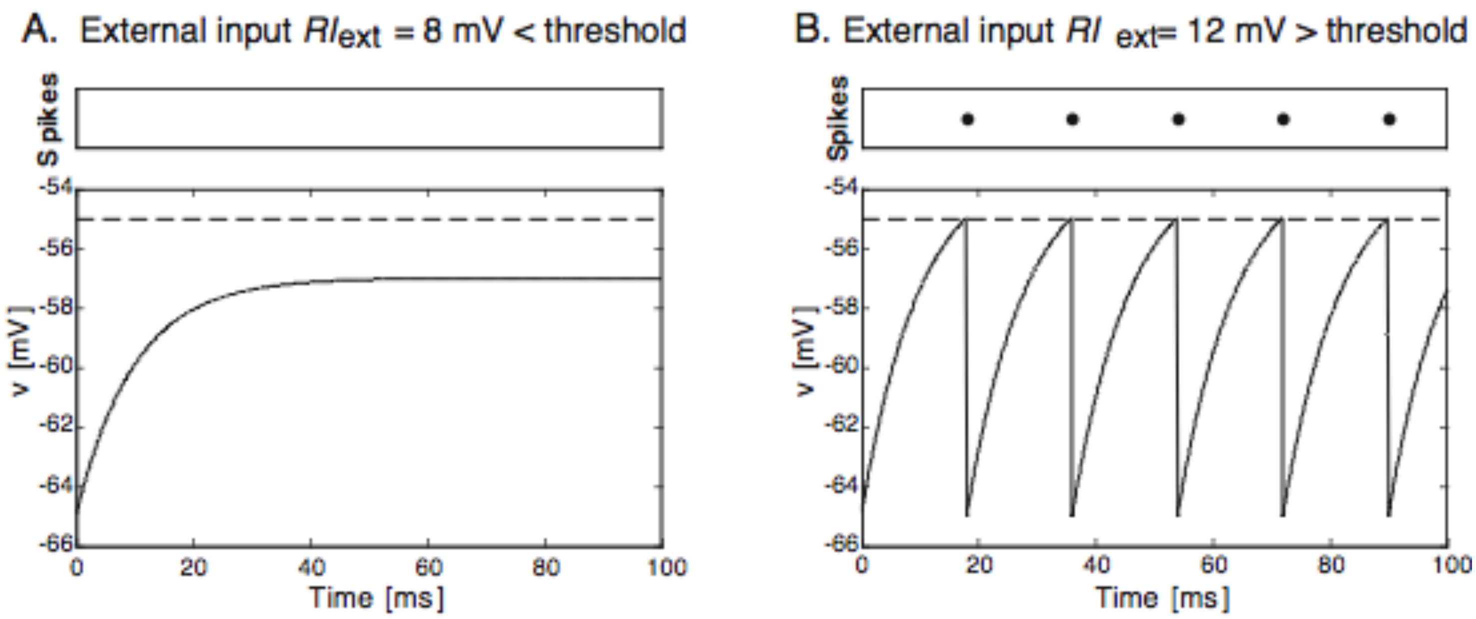
$$\tau_m \frac{dv(t)}{dt} = -(v(t) - E_L) + RI(t), \tag{1}$$

$$v(t^f) = \vartheta. \tag{2}$$

$$\lim_{\delta \to 0} v(t^f + \delta) = v_{res}, \tag{3}$$



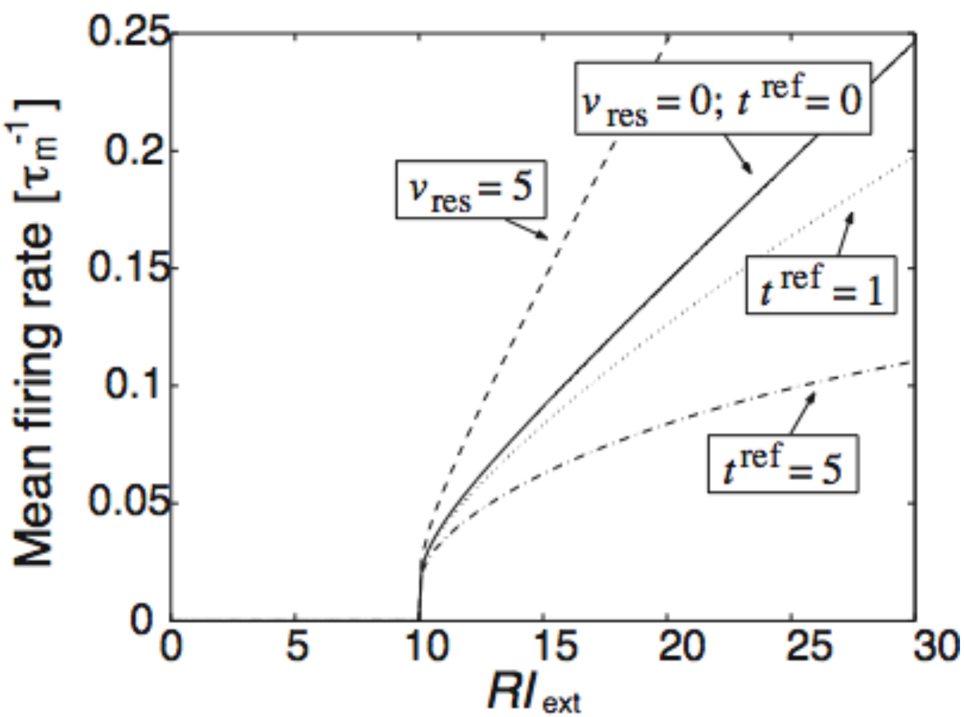
IF simulation



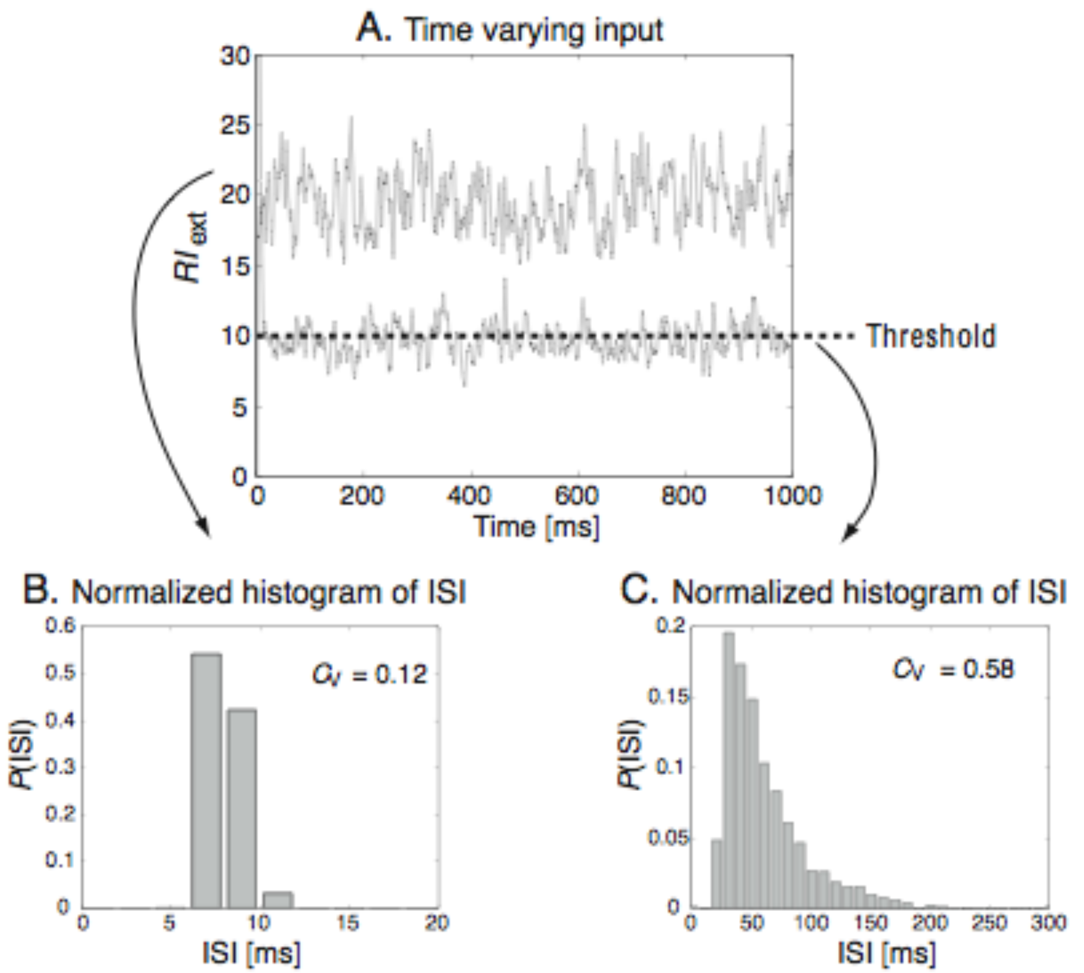
IF gain function

The inverse of the first passage time defines the firing rate:

$$\bar{r} = (t^{\mathrm{ref}} - \tau_{\mathrm{m}} \ln \frac{\vartheta - RI}{v_{\mathrm{res}} - RI})^{-1}$$



IF resistance to noise

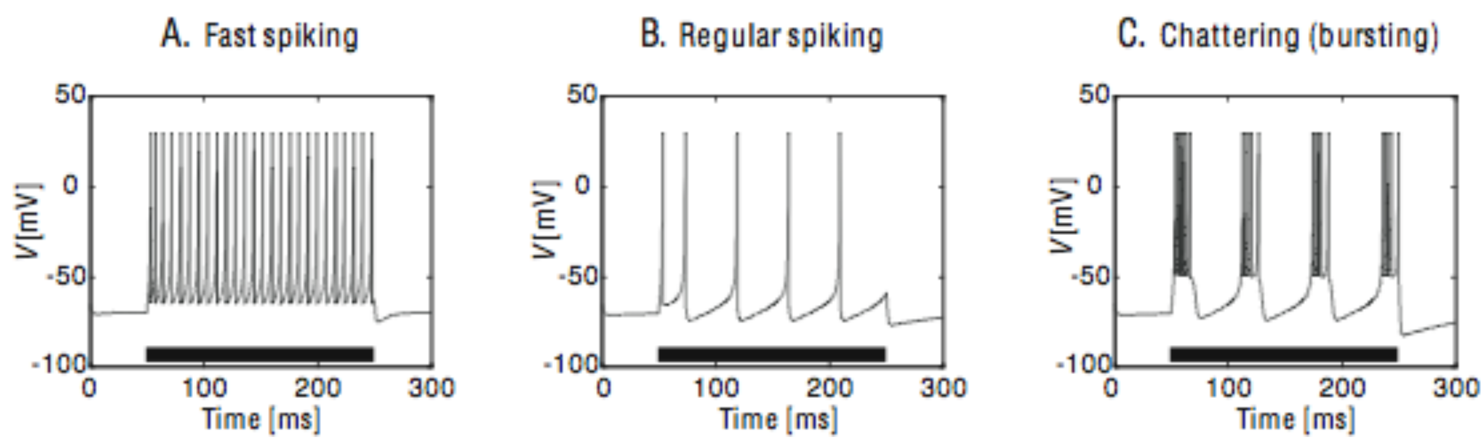


The Izhikevich neuron

$$\frac{\mathrm{d}v(t)}{\mathrm{d}t} = 0.04v^2(t) + 5v(t) + 140 - u + I(t)$$

$$\frac{\mathrm{d}u(t)}{\mathrm{d}t} = a(bv - u)$$

$$v(v > 30) = c \text{ and } u(v > 30) = u + d$$



The McCulloch-Pitts neuron

$$h = \sum_i x_i^{\text{in}}$$

$$x^{\text{out}} = \begin{cases} 1 & \text{if } h > \Theta \\ 0 & \text{otherwise} \end{cases}$$

BULLETIN OF
MATHEMATICAL BIOPHYSICS
VOLUME 5, 1943

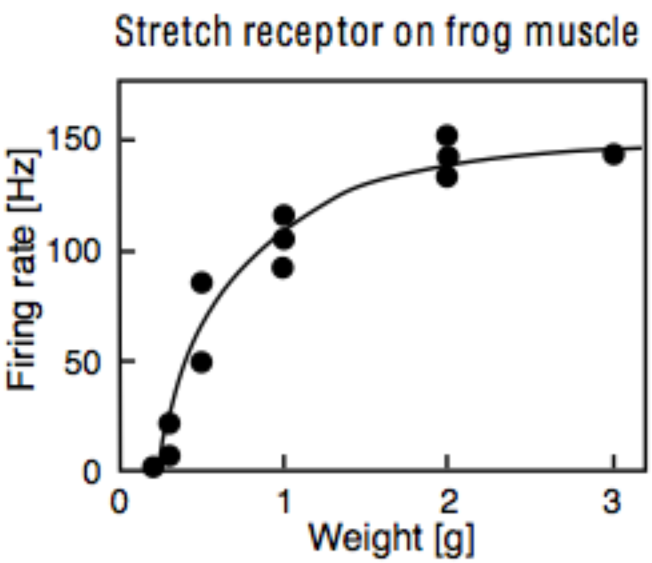
A LOGICAL CALCULUS OF THE IDEAS IMMANENT IN NERVOUS ACTIVITY

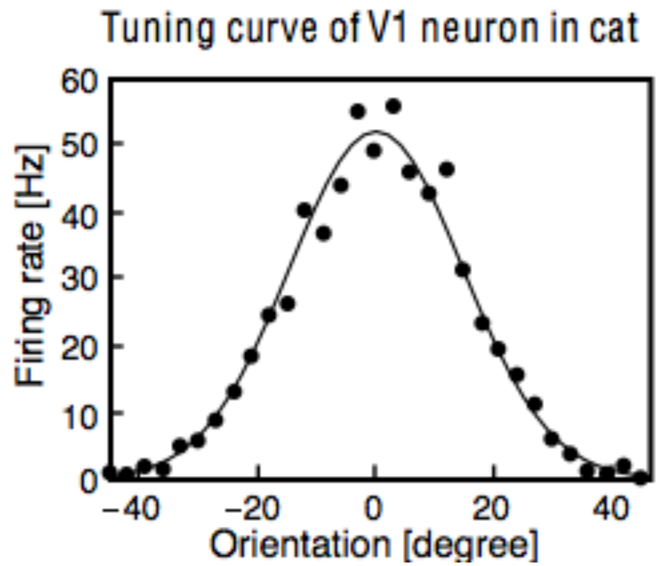
WARREN S. MCCULLOCH AND WALTER PITTS

FROM THE UNIVERSITY OF ILLINOIS, COLLEGE OF MEDICINE,
DEPARTMENT OF PSYCHIATRY AT THE ILLINOIS NEUROPSYCHIATRIC INSTITUTE,
AND THE UNIVERSITY OF CHICAGO

Because of the "all-or-none" character of nervous activity, neural events and the relations among them can be treated by means of propositional logic. It is found that the behavior of every net can be described in these terms, with the addition of more complicated logical means for nets containing circles; and that for any logical expression satisfying

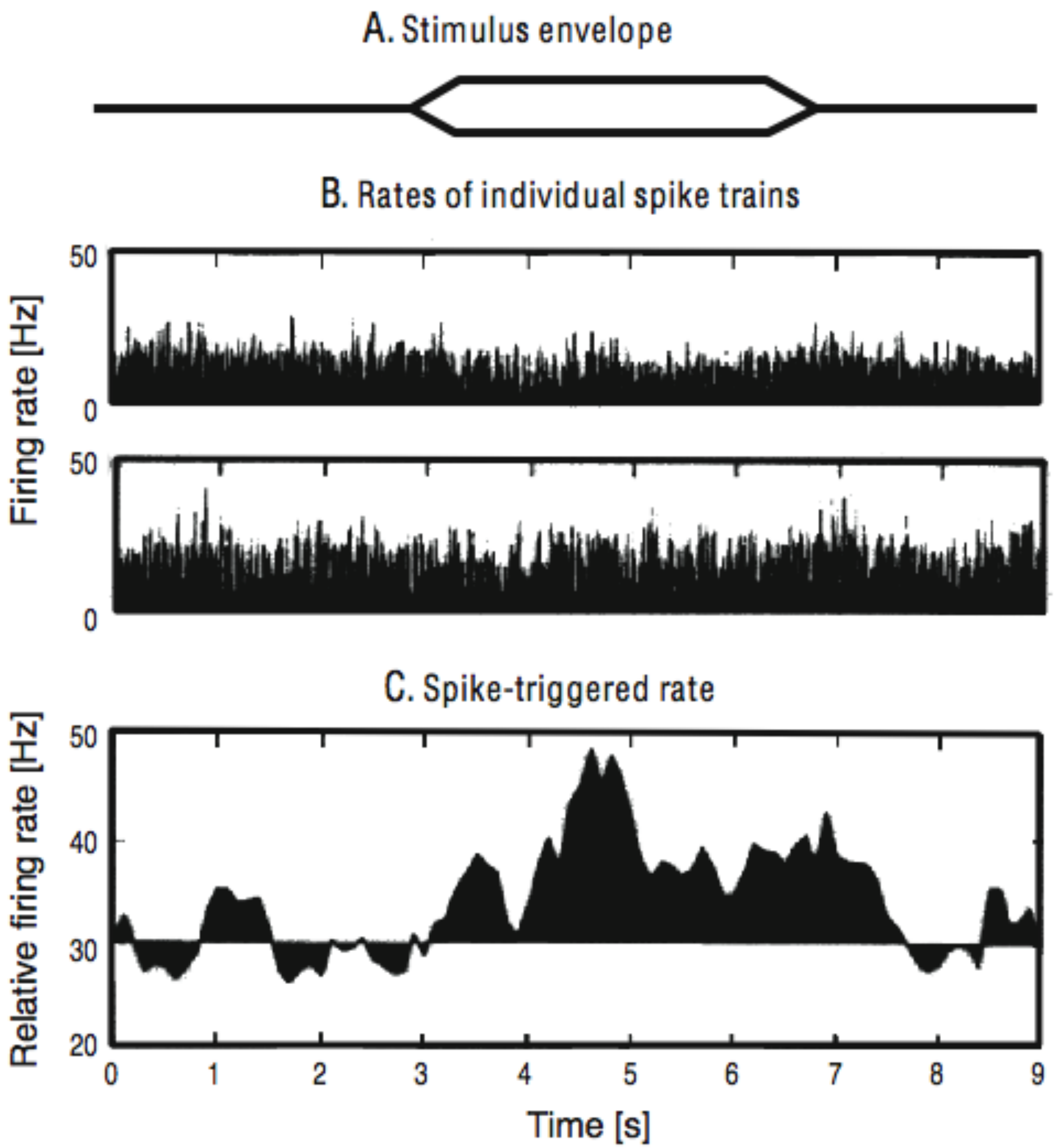
The firing rate hypothesis





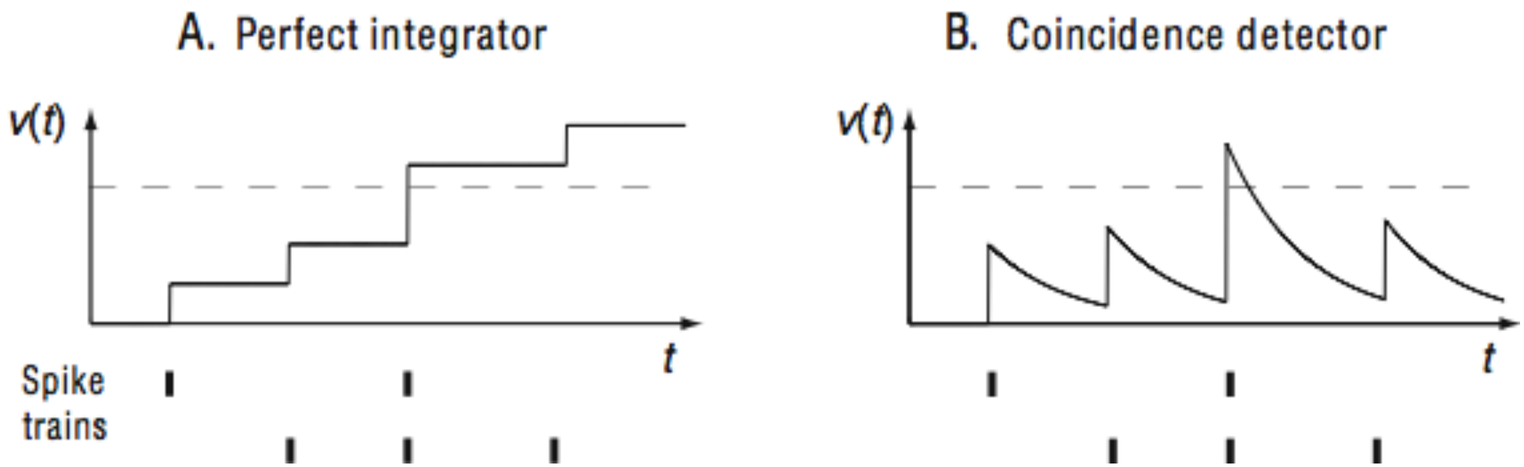
Edgar Adrian
The Nobel Prize in Physiology or Medicine 1932

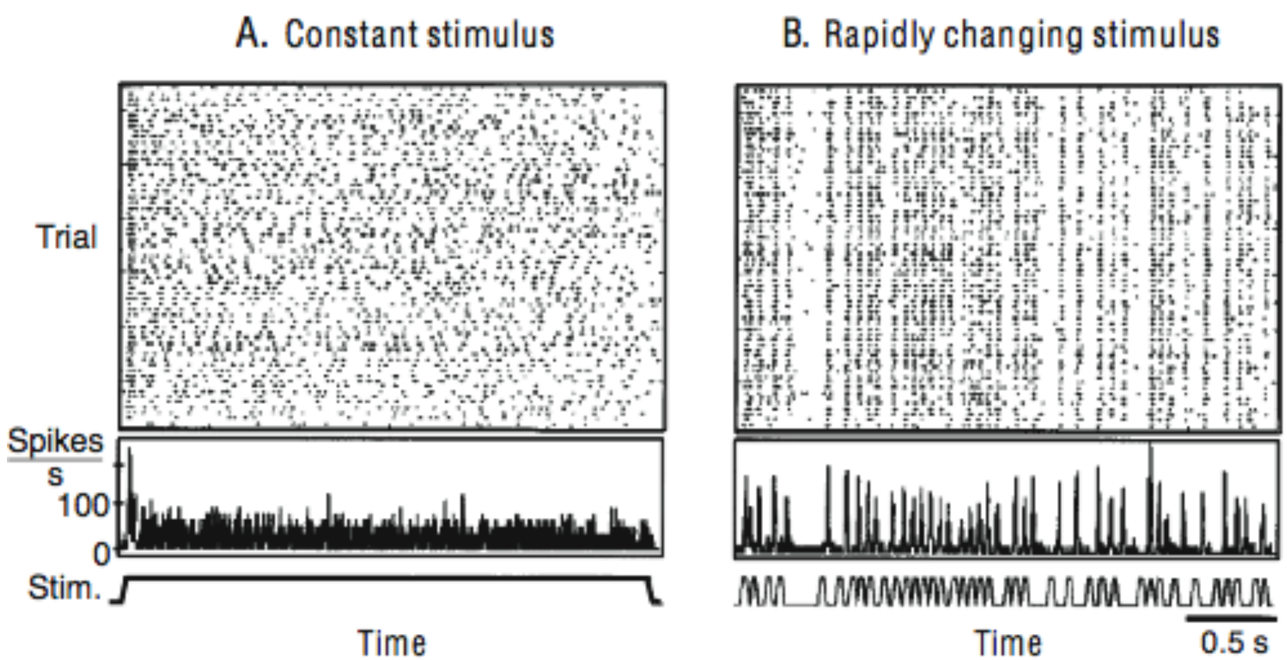
Counter example: correlation code (?)



From DeCharms and Merzenich 1996

Integrator or coincidence detector?

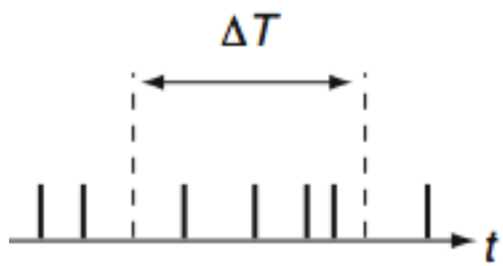




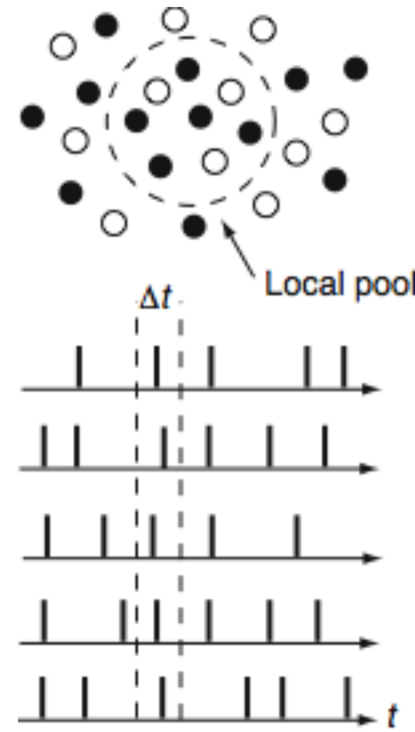
From Buracas et al. 1998

Population model

Temporal averaging



Population averaging



Population dynamics

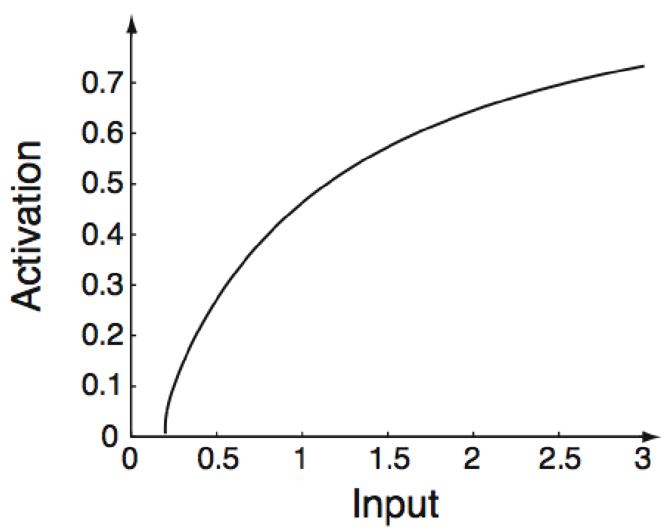
For slow varying input (adiabatic limit), when all nodes do practically the same, same input, etc (Wilson and Cowan,1972):

$$\tau \frac{\mathrm{d}A(t)}{\mathrm{d}t} = -A(t) + g(RI^{\mathrm{ext}}(t)).$$

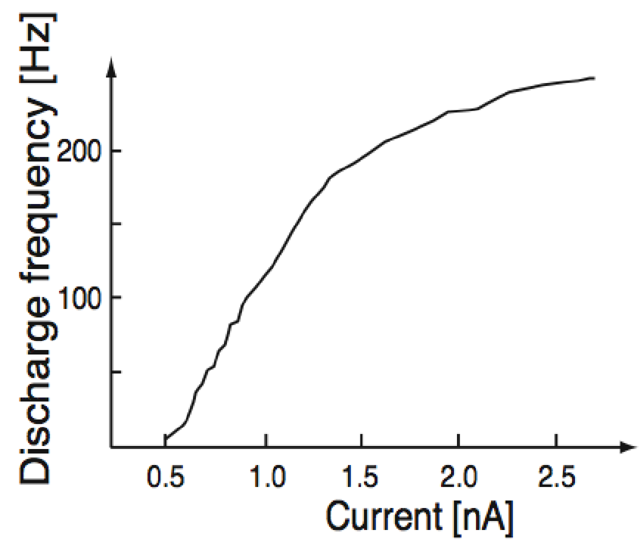
Gain function:

$$g(x) = \frac{1}{t^{\mathrm{ref}} - \tau \log(1 - \frac{1}{\tau x})},$$

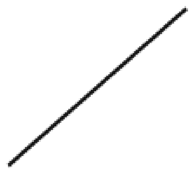
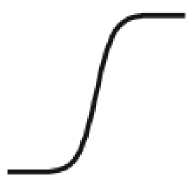
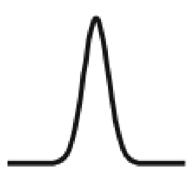
A. Activation function for population average in adiabatic limit



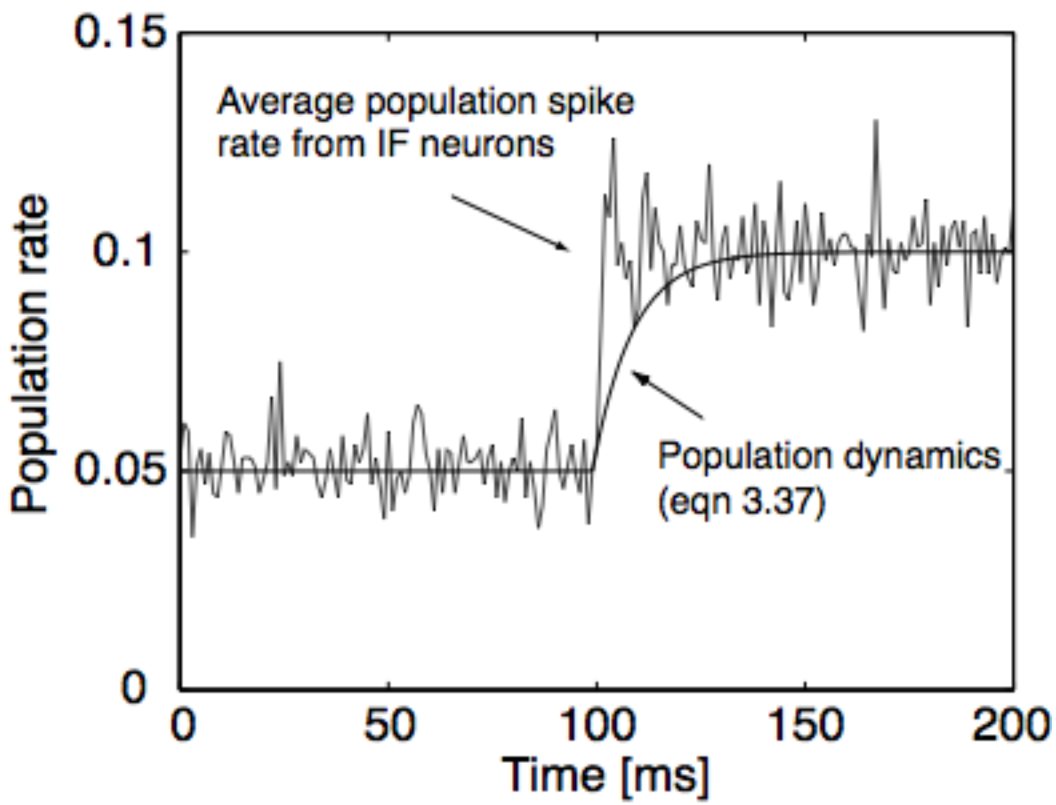
B. Activation function of hippocampal pyramidal neuron



Other gain functions

Type of function	Graphical represent.	Mathematical formula	MATLAB implementation
Linear		$g^{\text{lin}}(x) = x$	`x`
Step		$g^{\text{step}}(x) = \begin{cases} 1 & \text{if } x > 0 \\ 0 & \text{elsewhere} \end{cases}$	`floor(0.5*(1+sign(x)))`
Threshold-linear		$g^{\text{theta}}(x) = x\,\Theta(x)$	`x.*floor(0.5*(1+sign(x)))`
Sigmoid		$g^{\text{sig}}(x) = \frac{1}{1+\exp(-x)}$	`1./(1+exp(-x))`
Radial-basis		$g^{\text{gauss}}(x) = \exp(-x^2)$	`exp(-x.^2)`

Fast population response



Further readings

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Nicolas Brunel and Xiao-Jing Wang, (2001), **Effects of neuromodulation in a cortical network model of working memory dominated by recurrent inhibition**, in **Journal of Computational Neuroscience** 11: 63–85.