Assignment 3

Questions 1-4 is an individual assignment and Questions 5 and 6 is a group assignment. Submit to prof4155@cs.dal.ca with subject line A3a for the individual part and A3b for the group part by Monday October 1, 4pm

1. Plot a histogram of random numbers drawn from the Chi-square distribution and the Trappenberg distribution. The Trappenberg distribution is given by

\[ p(x) = \begin{cases} \ a_n \| \sin(x) \| & \text{for } 0 < x < n\pi/2 \\ 0 & \text{otherwise} \end{cases} \]  

for \( n = 5 \). What is the mean, variance, and skewness of these distributions?

2. Explain if the random variables \( X \) and \( Y \) are independent if their marginal distribution is \( p(x) = x + 3\log(x) \) and \( p(y) = 3y\log(y) \), and the joined distributions is \( p(x,y) = xy\log(x) + 3y\log(xy) \).

3. (From Thrun, Burgard and Fox, Probabilistic Robotics) A robot uses a sensor that can measure ranges from 0m to 3m. For simplicity, assume that the actual ranges are distributed uniformly in this interval. Unfortunately, the sensors can be faulty. When the sensor is faulty it constantly outputs a range below 1m, regardless of the actual range in the sensor’s measurement cone. We know that the prior probability for a sensor to be faulty is \( p = 0.01 \).

Suppose the robot queries its sensors \( N \) times, and every single time the measurement value is below 1m. What is the posterior probability of a sensor fault, for \( N = 1, 2, ..., 10 \). Formulate the corresponding probabilistic model.

4. Given are four Bernoulli distributed random variables \( X_1, X_2, X_3 \) and \( Y \). The conditional probability of random variables \( X_i \) on \( Y \) is given by \( p(x_i|y) = 0.2 \) and \( p(x_i|\neg y) = 0.6 \), and all \( x_i \) are conditionally independent of each other given \( Y \). The marginal probability of \( Y \) is \( p(y) = 0.3 \). What is the probability of \( Y \) given \( X_1 = \text{true} \) and \( X_2 = \text{true} \) and \( X_3 = \text{false} \)?

5. Use the light sensor to measure distances to a surface and derive a sensor model for this sensor. Provide a parametric form of your model and include estimations of the parameters.

6. Derive a motion model for the tribot when driving the motors with different power parameters. Provide a parametric form of your model and include estimations of the parameters.