The Basic Augmenting Paths Algorithm

CSCI 4113/6101

Along which Edges Can We Send Flow?

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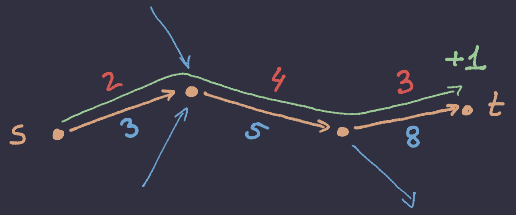
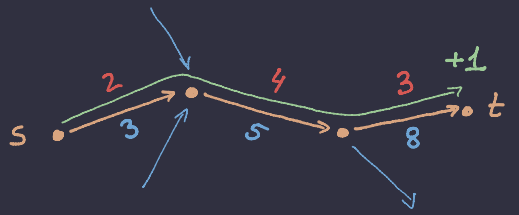
Edges that are not saturated

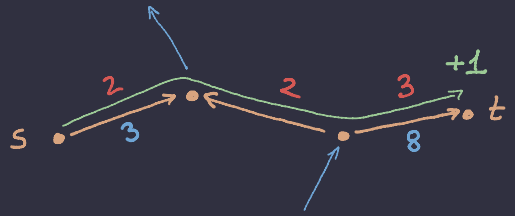


Along which Edges Can We Send Flow?

Edges that are not saturated



Edges that carry flow in the other direction ("Push flow back")



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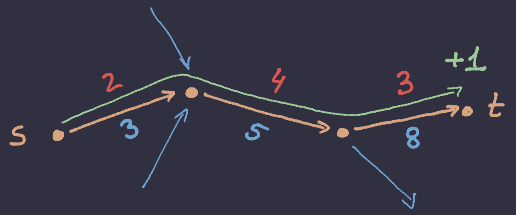

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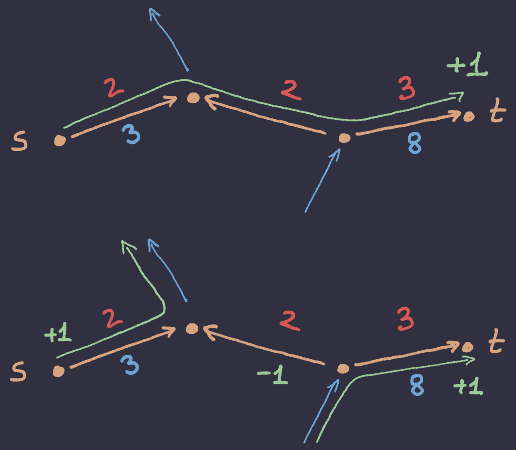
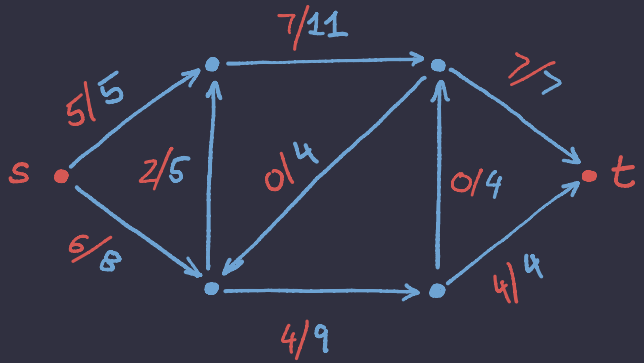


The Residual Network

Graph G, Flow f

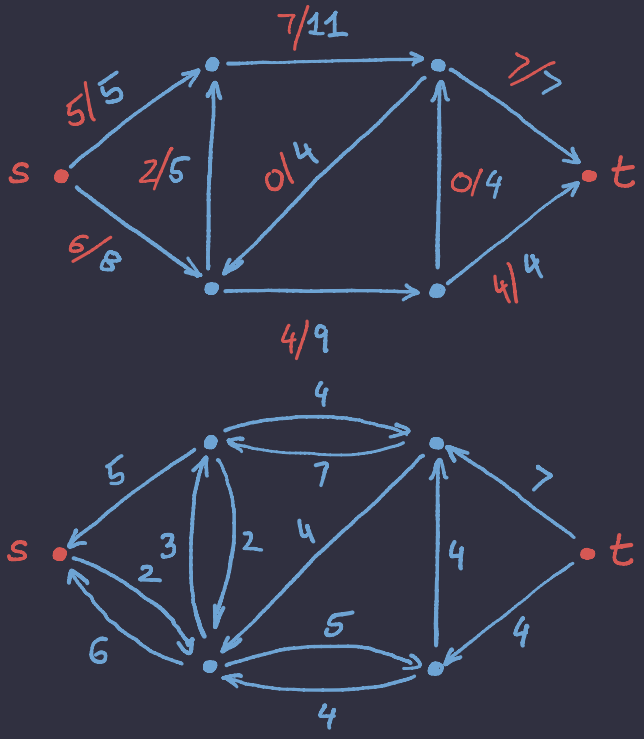


The Residual Network

Graph G, flow f

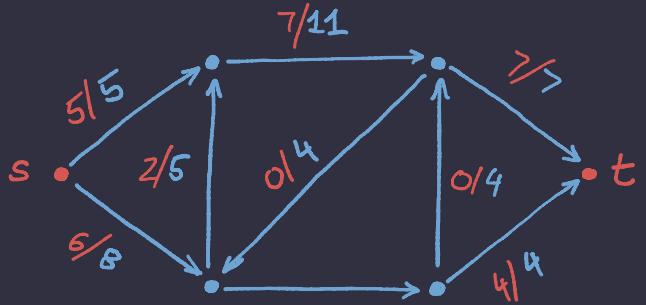
Residual network G_f

- Graph G_f
- Residual capacities c_f



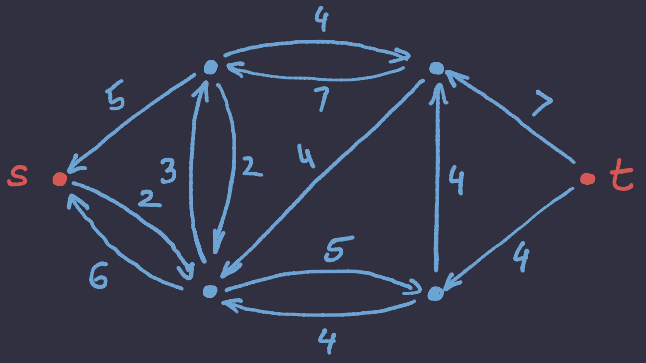
The Residual Network

Graph G, flow f



Residual network G_f

- Graph G_f
- Residual capacities c_f



Augmenting path = st-path in G_f

Ford-Fulkerson: The Basic Augmenting Paths Algorithm

Ford-Fulkerson (G, c):

$f(e) = 0 \quad \forall e \in G$

do forever

G_f = residual network of G and f

if $\exists$ st-path P in G_f then

$\delta = \min_{e \in P} c_f(e)$

$$\forall (u,v) \in G : f(u,v) = \begin{cases} f(u,v) + \delta & \text{if } (u,v) \in P \\ f(u,v) - \delta & \text{if } (v,u) \in P \\ f(u,v) & \text{otherwise} \end{cases}$$

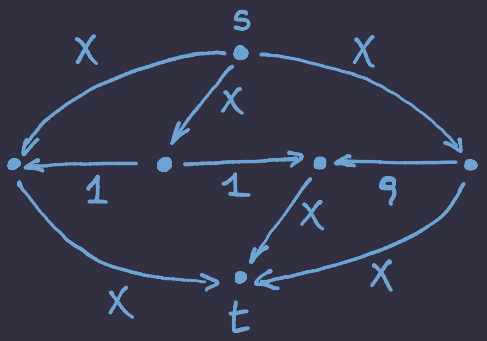
else

return f

Ford-Fulkerson May Not Terminate

$$q = \frac{\sqrt{5}-1}{2} \quad \Rightarrow \quad q^2 + q = 1$$

X = large



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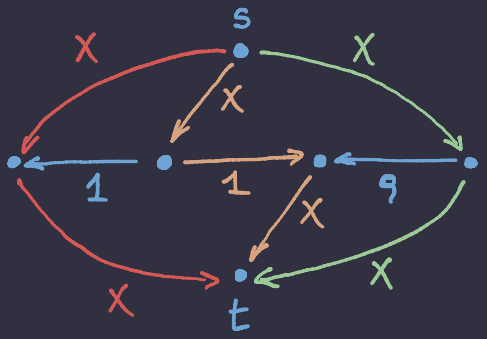
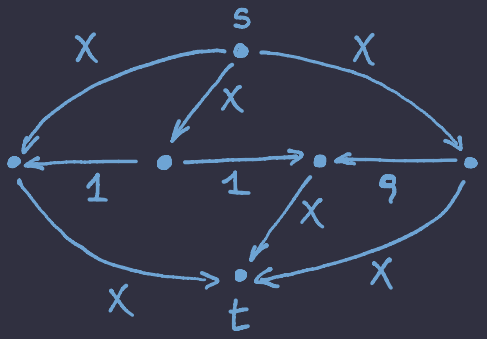


$$q = \frac{\sqrt{5}-1}{2} \implies q^2 + q = 1$$

X = large

Maximum flow $= 2X + 1$

Ford-Fulkerson May Not Terminate



$q = \frac{\sqrt{5}-1}{2} \Rightarrow q^2 + q = 1$

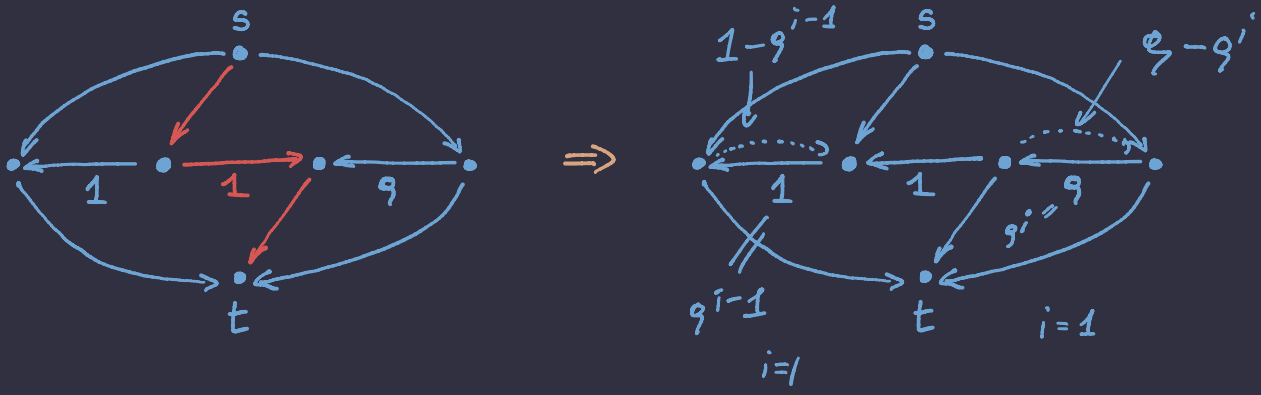
X = large

Maximum flow $= 2X + 1$

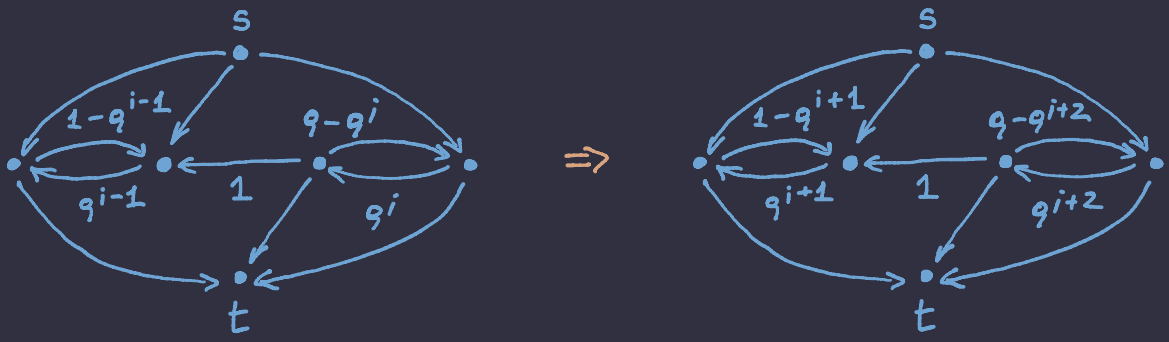
Ford-Fulkerson may

- Run forever
- Converge to flow $1 + \sum_{i=1}^{\infty} 2q^i = \frac{4}{3-\sqrt{5}} - 1 < 5$

The First Augmentation



The Endless Loop



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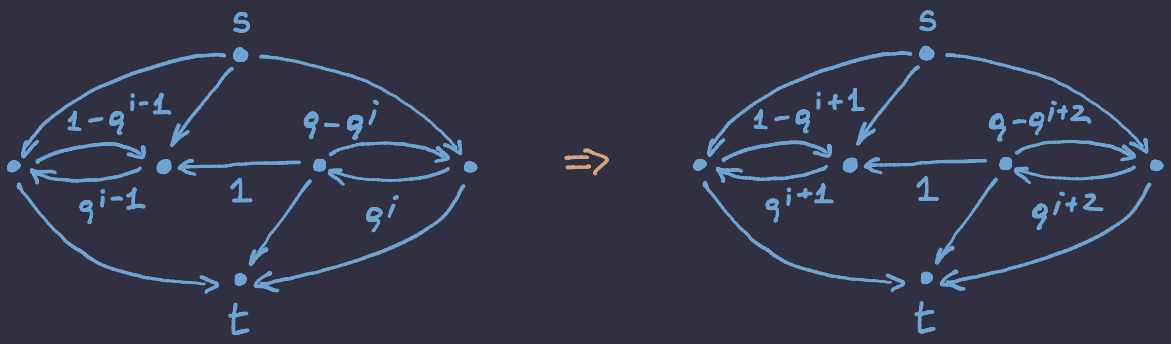
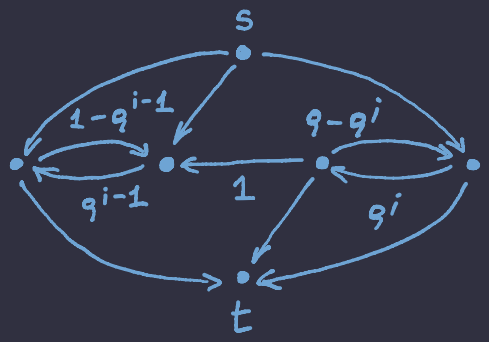
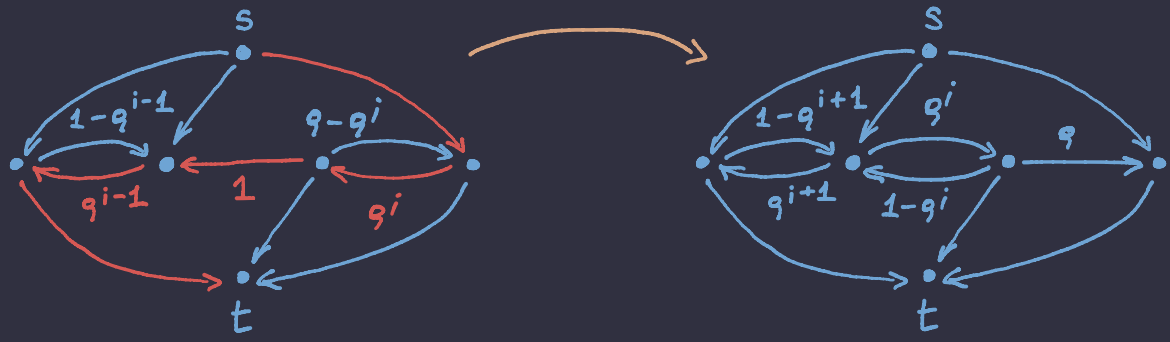
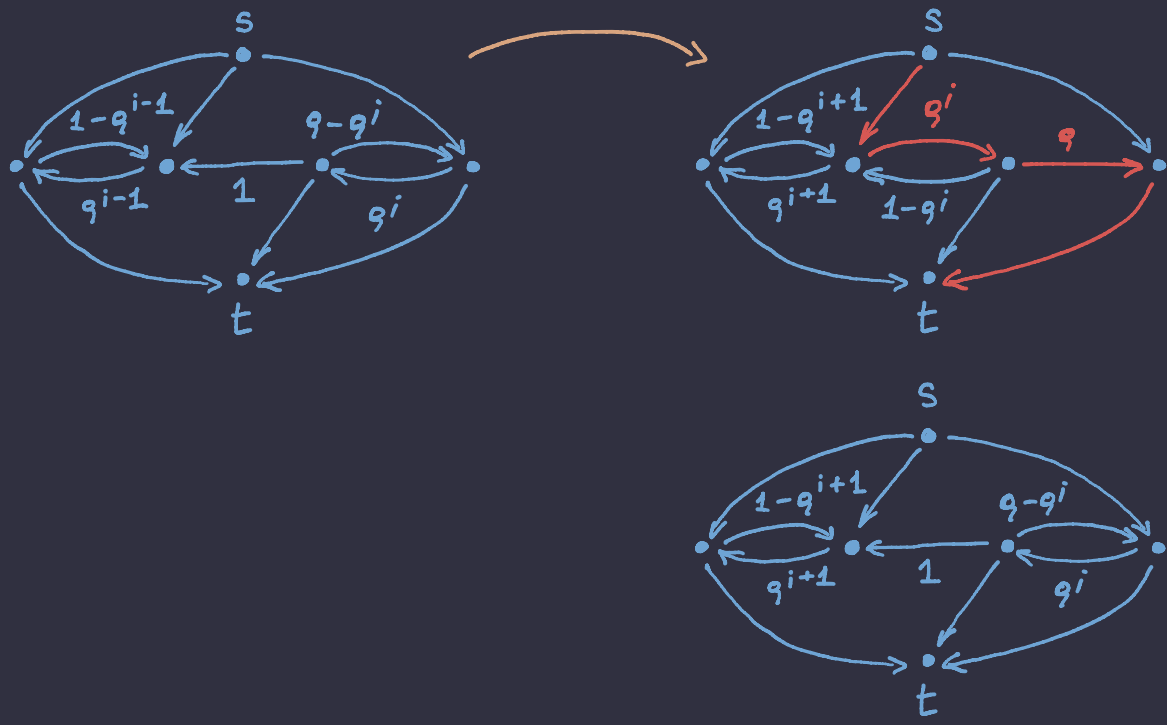


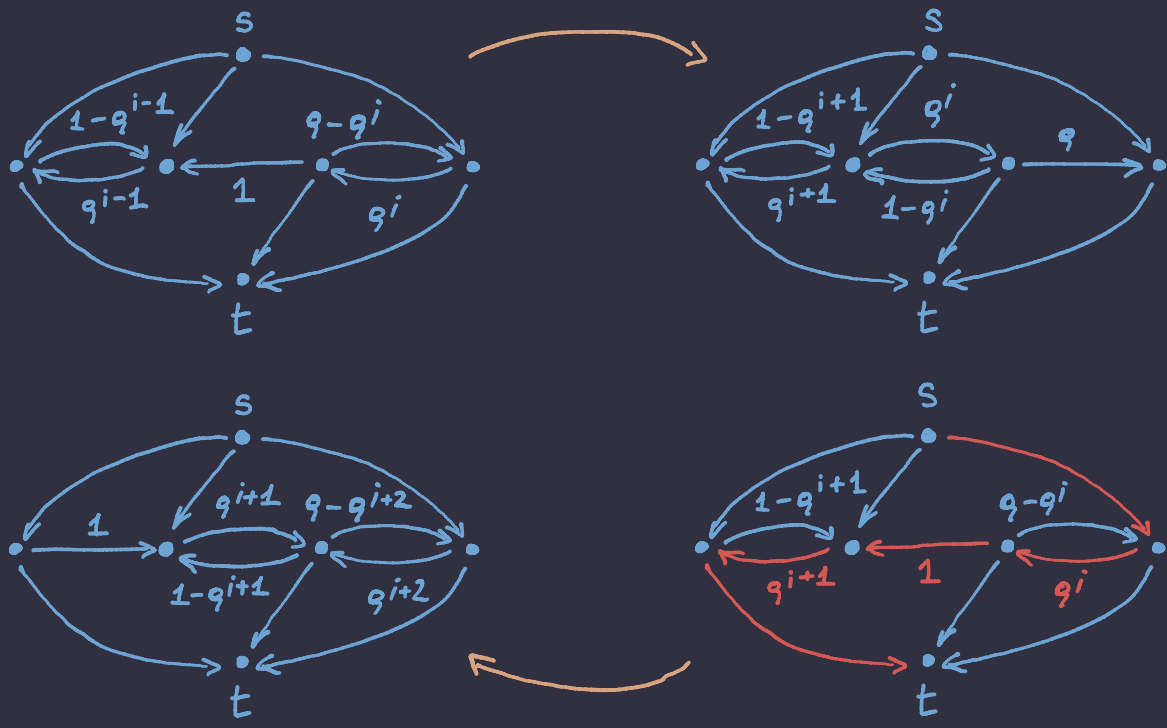
Each iteration pushes $2(q^i + q^{i+1})$ units of flow from s to t

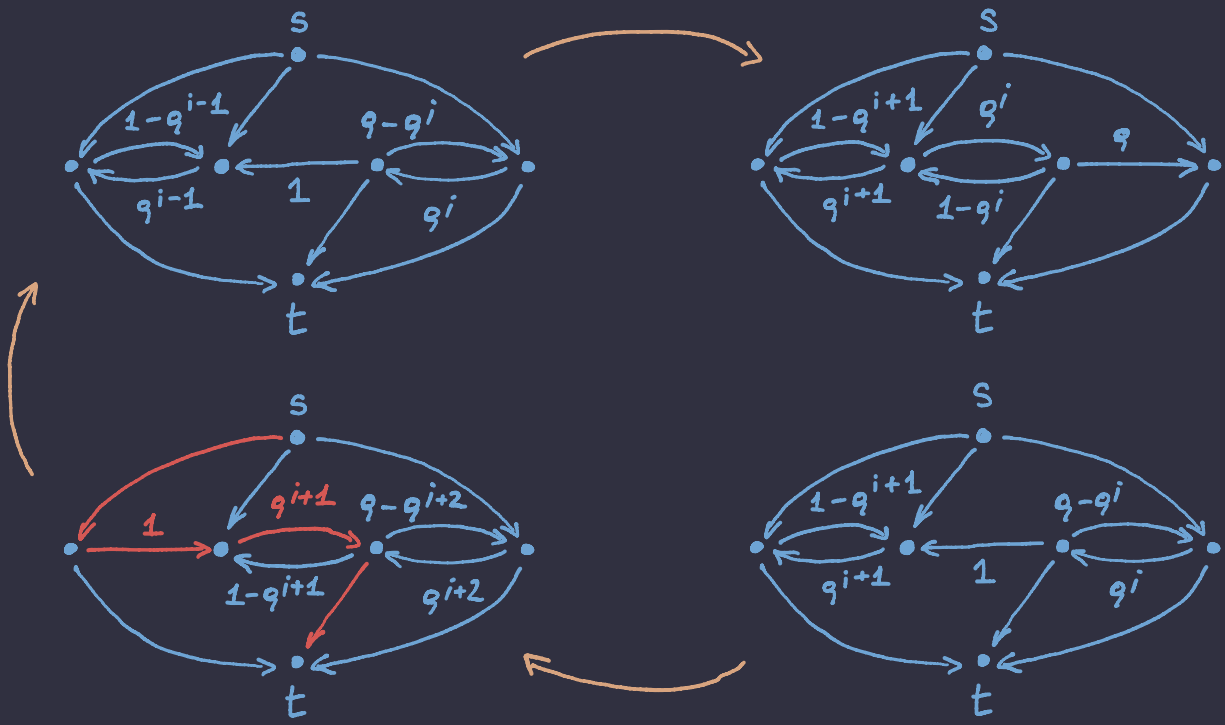
$$\Rightarrow \text{Total} = 1 + \sum_{i=1}^{\infty} 2q^i = \frac{4}{3-\sqrt{5}} - 1$$


s
$1-q^{i-1}$
$q-q^i$
q^{i-1}
1
q^i
t


s
$1-q^{i-1}$
$q-q^i$
q^{i-1}
1
q^i
t
s
$1-q^{i+1}$
q^i
q
q^{i+1}
$1-q^i$
t


s
$1-q^{i-1}$
$q-q^i$
q^{i-1}
1
q^i
t
s
$1-q^{i+1}$
q^i
q
q^{i+1}
$1-q^i$
t
s
$1-q^{i+1}$
$q-q^i$
q^{i+1}
1
q^i
t


s
t
$1-q^{i-1}$
$q-q^i$
q^{i-1}
1
q^i
$1-q^{i+1}$
q^i
q
q^{i+1}
$1-q^i$
1
q^{i+1}
$q-q^{i+2}$
$1-q^{i+1}$
q^{i+2}
$1-q^{i+1}$
$q-q^i$
q^{i+1}
1
q^i


s
t
$1-q^{i-1}$
$q-q^i$
q^{i-1}
1
q^i
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q
q^{i+1}
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q^{i+2}
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$q-q^i$
q^{i+1}
1
q^i