Sample Solution

Assignment 2

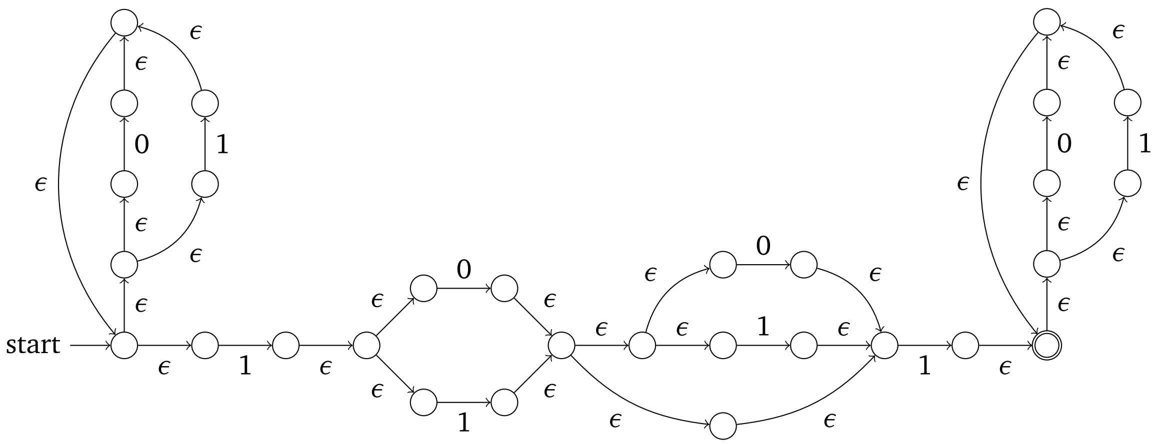
CSCI 3136 — Winter 2018

Question 1

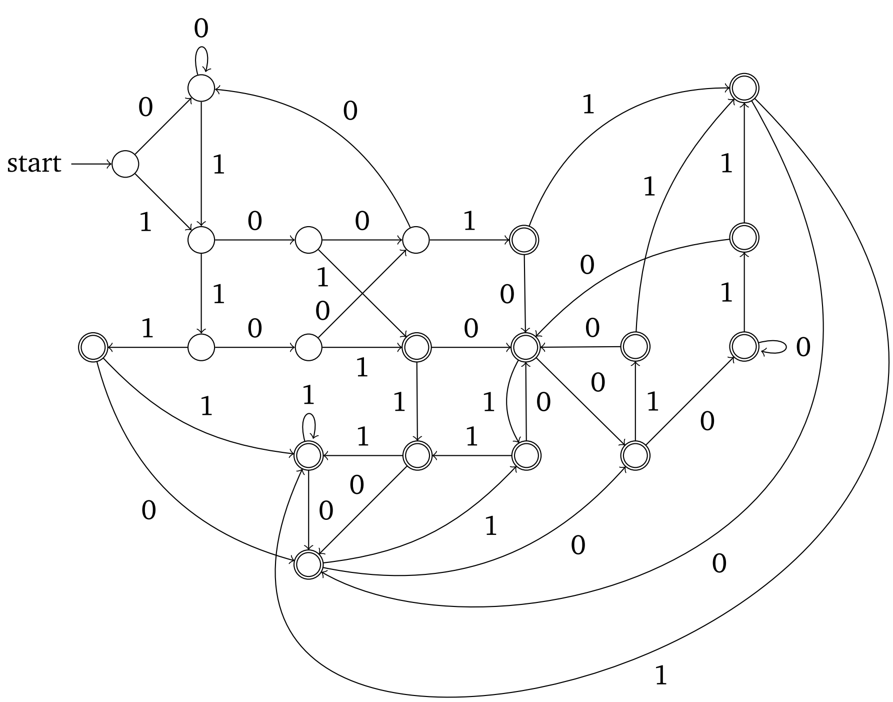
(a) Regular expression:

$$R = \texttt{.*1.(.|}\epsilon\texttt{)1.*}$$

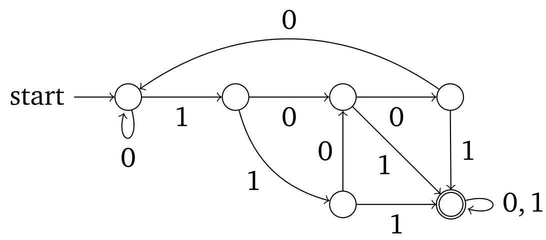
NFA:



DFA:



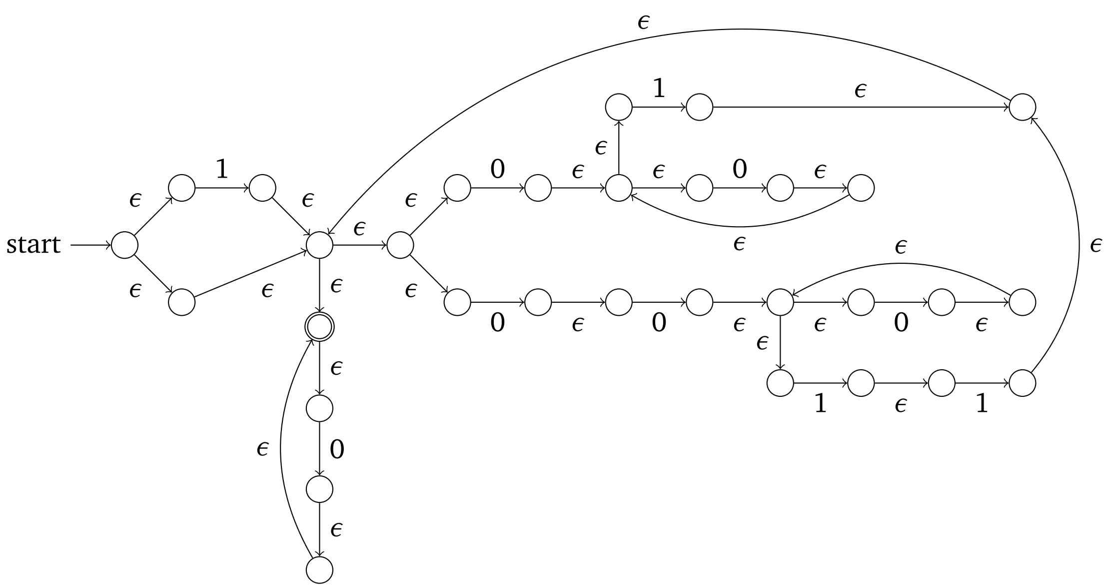
Minimized DFA:



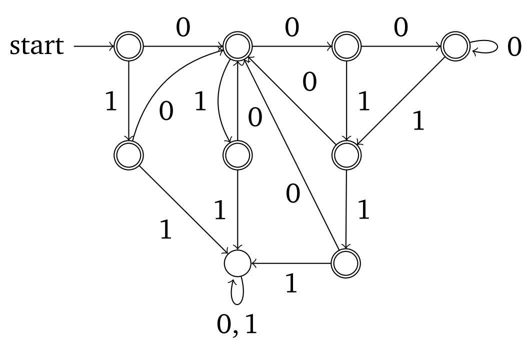
(b) Regular expression:

$$R = (1|\epsilon)(00{*}1|000{*}11){*}0{*}$$

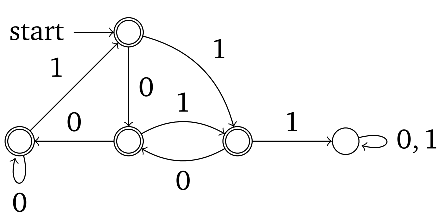
NFA:



DFA:



Minimized DFA:



Question 2

(a) Assume this language is regular. Then, by the Pumping Lemma, there exists a constant $n_{\mathscr{L}}$ such that every $\sigma \in \mathscr{L}$ with $|\sigma| \geq n_{\mathscr{L}}$ can be written as the concatenation of three strings α, β, and γ such that $|\alpha\beta| \leq n_{\mathscr{L}}$, $|\beta| > 0$, and $\alpha\beta^k\gamma \in \mathscr{L}$ for all $k \geq 0$.

So consider the string $\sigma = 1^{n_{\mathscr{L}}}0^{n_{\mathscr{L}}} \in \mathscr{L}$. Since $|\sigma| > n_{\mathscr{L}}$, we have $\sigma = \alpha\beta\gamma$ with α, β, γ satisfying the conditions above. In particular, since $|\alpha\beta| \leq n_{\mathscr{L}}$ and the first $n_{\mathscr{L}}$ characters of σ are all 1s, we have $\alpha = 1^{\ell}$ and $\beta = 1^m$ with $\ell + m \leq n_{\mathscr{L}}$ and $m > 0$. By the Pumping Lemma, $\alpha\beta\beta\gamma \in \mathscr{L}$. This, however, is a contradiction because $\alpha\beta\beta\gamma = 1^{n_{\mathscr{L}}+m}0^{n_{\mathscr{L}}}$, that is, it has more 1s than 0s.

(b) Assume this language is regular. Then, by the Pumping Lemma, there exists a constant $n_{\mathscr{L}}$ such that every $\sigma \in \mathscr{L}$ with $|\sigma| \geq n_{\mathscr{L}}$ can be written as the concatenation of three strings α, β, and γ such that $|\alpha\beta| \leq n_{\mathscr{L}}$, $|\beta| > 0$, and $\alpha\beta^k\gamma \in \mathscr{L}$ for all $k \geq 0$.

So consider any string $\sigma \in \mathscr{L}$ with $|\sigma| = 2^m > n_{\mathscr{L}}$. Since $|\sigma| > n_{\mathscr{L}}$, we have $\sigma = \alpha\beta\gamma$ with α, β, γ satisfying the conditions above. In particular, $\sigma' = \alpha\beta\beta\gamma \in \mathscr{L}$. We have $|\sigma'| = 2^m + t$, where $t = |\beta| > 0$. Since $|\alpha\beta| \leq n_{\mathscr{L}} < 2^m$, we have in particular that $t = |\beta| < 2^m$. Since $\sigma' \in \mathscr{L}$, we have $|\sigma'| = 2^{m'}$ for some integer $m' \geq 0$, that is, $2^m + t = 2^{m'}$. However, since $t > 0$, we have $2^m + t > 2^m$; since $t < 2^m$, we have $2^m + t < 2^{m+1}$. Thus, no such integer exists, a contradiction.