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ACOUSTICAL DIFFRACTION MODELING UTILIZING THE HUYGENS-FRESNEL PRINCIPLE

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Abstract – *This paper describes the application of the Huygens-Fresnel Principle to acoustical diffraction modeling. A theoretical formulation of the optics-based Huygens-Fresnel Principle is presented followed by a discussion regarding the modifications necessary to apply the Huygens-Fresnel Principle to acoustical diffraction modeling. Experimental results indicate the method is capable of modeling acoustical diffraction phenomena in a simple and efficient manner, making it attractive for interactive virtual environments.*

Keywords – *acoustical diffraction, Huygens-Fresnel, virtual environment*

I. INTRODUCTION

Diffraction of sound refers to the "bending mode" of sound propagation whereby sound waves go ("bend") around an obstacle that lies directly in the line of straight propagation between the sound source and receiver [1], allowing us to hear sounds around corners and barriers. Diffraction is dependent on both wavelength and obstacle/surface size, increasing as the ratio between wavelength and obstacle size is increased [1]. The frequency spectrum of audible sound ranges from approximately 20Hz to 20kHz, corresponding to wavelengths ranging from 0.02m to 17m. Since the dimensions of many of the objects/surfaces encountered in our daily life are within an order of magnitude as the wavelength of audible sounds, diffraction is an elementary means of sound propagation, especially when there is no direct path between the sound source and the receiver [2]. Many auralization methods, and in particular those intended for virtual environment applications where dynamic update rates are necessary, are based on geometric acoustics. They assume sound is a ray phenomena [1] and model all interactions between a sound ray and objects/surfaces as specular, thus ignoring important effects such as diffraction and diffusion. Although ray based approaches are simple to model and implement, they are valid primarily for higher frequency sounds where reflections are indeed primarily specular. In addition, they typically ignore the wavelength of sound and any phenomena associated with it, including diffraction [3]. However, failure to account for diffraction can lead to a non-realistic auditory simulation.

Despite its importance, diffraction is typically ignored by many geometrical acoustical techniques [4], [2]. That being said, a limited number of research efforts have investigated acoustical diffraction modeling for virtual environment applications. Tsingos et al. [2] describe an extension to a beam tracing approach capable of approximating diffraction. Their frequency domain method, which is based on the *uniform theory of diffraction* (UTD) [5], is valid primarily for higher frequencies. Validation of their approach is shown in [6] and involves a comparison between the actual measured impulse response in a simple enclosure (the "Bell Labs Box") and the impulse response obtained by simulating the enclosure. Their technique was the first use of a physically based diffraction model to produce interactive rate sounds in a complex virtual environment.

Various other research efforts have examined non-geometric acoustics based diffraction modeling. Torres et al. [4] describe a time-domain model based on the Biot-Tolstoy-Medwin technique [7], which computes edge diffraction components and combinations of specular and diffracted components. Lokki et al. [8] and Svensson et al. [9] have also investigated diffraction modeling based on the Biot-Tolstoy-Medwin technique. Such techniques are currently not applicable to interactive applications due to complexity issues.

Diffraction is a wave phenomenon and as a result, it is inherent in both sound and light waves (in addition to other waves as well). With respect to light, diffraction has received considerable attention. Diffraction effects are an important aspect in the field of optics and have been studied for hundreds of years and several prevailing theories have been established. One such theory is the Huygens-Fresnel Principle, originally formulated by Christian Huygens in 1678 and later modified/extended by Augustin Fresnel. Although the Huygens-Fresnel Principle is a rather simple approach however it can satisfactorily describe a large number of diffraction configurations in a simple and efficient manner. Furthermore, as shown by Kirchoff, the

Huygens-Fresnel Principle is directly derivable form the scalar differential wave equation [10]. The Huygens-Fresnel Principle is based on the assumption that at every time instant, every point on a primary wavefront can be thought of as a continuous emitter of secondary wavelets (sources) and these secondary wavelets combine to produce a new wavefront in the direction of propagation. This assumption fits nicely with particle/ray based auralization methods whereby the acoustics of an environment is determined by emitting sound "particles" from a sound source and tracing them through the environment.

This paper describes a simple and efficient probabilistic acoustical diffraction modeling technique based on the Huygens-Fresnel Principle. The technique described here is incorporated into the sonel mapping framework [11]. The goal of sonel mapping is to model the acoustics of an environment, taking into account the relevant acoustical phenomena experienced by a propagating sound in an efficient manner at interactive rates for use in dynamic virtual environments.

A. Paper Organization

The remainder of the paper is organized as follows. A brief introduction to the sonel mapping method is provided in Section II. An introduction to the Huygens-Fresnel Principle is provided in Section III while its application to acoustical modeling is described in Section IV. Experimental results are provided in Section V and finally, a summary, discussion of future work and concluding remarks are given in Section VI.

II. SONEL MAPPING

Sonel mapping [11] is an application of the photon mapping image synthesis method [12] to auralization. Sonel mapping is a two-pass probabilistic, "particle-based", acoustical modeling method whose goal is to model the propagation of sound within an environment, currently taking into consideration both specular and diffuse reflections and absorption in an efficient manner.

In the first pass (the *sonel tracing* stage), sound elements known as *sonels* are emitted from each sound source and traced through the scene until they interact with a surface. Each sonel can be viewed as a packet of information propagating from the sound source to the receiver, carrying the relevant information required to simulate mechanical wave propagation. Upon encountering a surface, a Russian Roulette strategy is used to determine the type of interaction between the sonel and the surface [13]. Based on the characteristics of the surface and the result of a randomly generated number, the sonel is reflected either specularly, diffusely or is completely absorbed by the surface. When the interaction is diffuse reflection at a point p, the sonel is stored in a structure called a *sonel map* while a "new" sonel is generated and reflected diffusely by choosing a random direction over the hemisphere centered about p. Upon encountering a specular surface, the sonel is reflected specularly where the angle of reflection is equal to the angle of incidence (specularly reflected sonels are not stored).

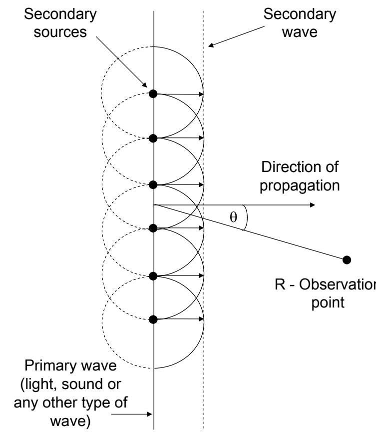

Fig. 1. Huygens' Principle.

In the second stage (the *rendering* stage), the room impulse response is estimated through the use of the previously constructed sonel map coupled with acoustic distribution ray tracing. The impulse response is estimated by emitting acoustic rays from each receiver and tracing them through the scene while recording their interaction with any objects/surfaces. As in the sonel tracing stage, a Russian Roulette strategy is used to determine the type of interaction between an acoustic ray and a surface it encounters. When the interaction at point p is a diffuse reflection, the acoustic ray is terminated and the sonel map is used to provide an estimate of the sound energy leaving point p and arriving at the receiver using a *density estimation* algorithm. When the interaction is a specular reflection, as in the sonel tracing stage, the sonel is reflected specularly such that the angle of reflection equals the angle of incidence. However, in contrast to stage one, when a sound ray encounters a sound source, its energy is scaled to account for attenuation by the medium and added to the accumulating impulse response. As in stage one, when the interaction is determined to be absorption, the sonel is terminated. Other acoustical effects can be modeled using this framework. This paper for example, extends the basic approach to handle diffraction.

III. INTRODUCTION TO THE HUYGENS-FRESNEL PRINCIPLE

The Huygens Principle, developed by Christian Huygens in 1678, is based on the wave theory of light. Referring to Figure 1, the Huygens Principle states that every point on the primary wavefront can be thought of as a continuous emitter of secondary wavelets (sources) and these secondary wavelets combine to produce a new wavefront in the direction of propagation [14].

The Huygens principle is itself not completely correct since if each of the secondary wavelets were emitted uniformly in all

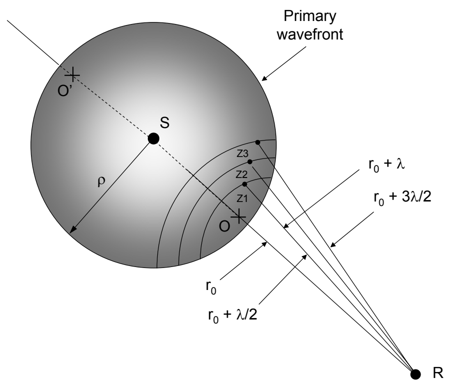

Fig. 2. Fresnel zones.

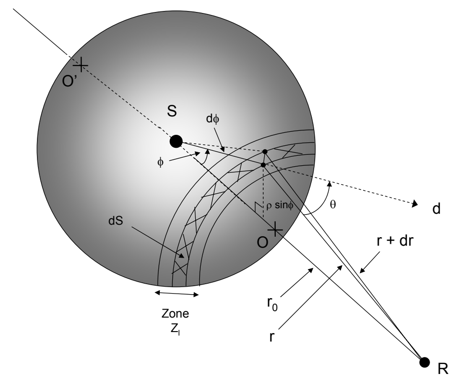

Fig. 3. Fresnel zone geometry.

directions then in addition to the forward propagating wavefront, a wavefront propagating in the reverse direction would also be observed when in fact it is not. As inferred by Fresnel and later formulated by Kirchoff, these secondary wavelets are emitted in a direction dependent manner based on an *obliquity* or *inclination* factor $K(\theta)$ as [10]

$$K(\theta) = \frac{1}{2}(1 + \cos(\theta)) \tag{1}$$

where, as illustrated in Figure 1, θ is the angle made with the normal to the primary wavefront d. The Huygens Principle and Fresnel's modification are collectively known as the Huygens-Fresnel Principle and can describe various diffraction configurations. A brief summary of the derivation of the Huygens-Fresnel Principle as it appears in [10] is provided for completeness in the following section.

A. Fresnel Zones

Referring to Figure 2, consider a sound source (S) and receiver (R) in free space (e.g., no obstacles between them). Having originated at S at time $t = 0$ with an amplitude E_0, at time t' the wave will have propagated a distance ρ and its amplitude will be E_0/ρ. The wavefront at time t' can be described as

$$E = \frac{E_0}{\rho} cos(\omega t' - k\rho) \tag{2}$$

where, $\omega = 2\pi f$ is the angular frequency and $k = 2\pi\lambda$ is the wave-number (λ is the wavelength). This expanding wavefront is divided into a number of ring-like regions, collectively known as the *Fresnel zones* [10]. The boundary of the n^{th} Fresnel zone corresponds to the intersection of the wavefront with a sphere of radius $r_0 + n\lambda/2$ centered at the receiver where, r_0 is equal to the distance between the receiver and the expanding wavefront after it has traversed a distance of ρ from the sound source. In other words, the distance from the receiver to each adjacent zone, differs by half a wavelength ($\lambda/2$).

Each Fresnel zone is finite in extent and therefore, as illustrated in Figure 3, a differential ring-shaped area dS can be defined within a zone. The secondary sources (wavelets) within dS are coherent and assumed to emit in phase with the primary wave. The secondary sources travel a distance r to reach the receiver at a time t, all of them arriving there with the same phase $\omega t - k(\rho + r)$. The strength of the secondary sources per unit area on dS, denoted by E_A, is proportional to E_0/ρ within a constant factor Q (e.g., $E_A = QE_0/\rho$, where $Q = 1/\lambda$).

The energy dE reaching the receiver from all the secondary sources on dS is given as

$$dE = K(\theta)\frac{E_A}{r}\cos[\omega t - k(\rho + r)]dS \tag{3}$$

where the obliquity factor $K(\theta)$ is assumed to be constant throughout dS and throughout the entire Fresnel zone. dS itself can be given as a function of r. Referring to Figure 3,

$$dS = \rho d\varphi 2\pi(\rho \sin\varphi). \tag{4}$$

Applying the law of cosines yields

$$r^2 = \rho^2 + (\rho + r_0)^2 - 2\rho(\rho + r_0)\cos\varphi. \tag{5}$$

Keeping ρ and r_0 constant and differentiating Equation 5 above gives

$$2rdr = 2\rho(\rho + r_0)\sin\varphi d\varphi. \tag{6}$$

Rearranging Equation 6 above, $d\varphi$ can be expressed as

$$d\varphi = \frac{2rdr}{2\rho(\rho + r_0)\sin\phi} \tag{7}$$

and using the value of $d\varphi$, dS can now be expressed as

$$dS = 2\pi\frac{\rho}{(\rho + r_0)}rdr. \tag{8}$$

Finally, the energy E_l arriving at the receiver from the l^{th} zone can then be determined by integrating over all differential areas across Z_l and is given as

$$E_l = K_l(\theta)2\pi\frac{E_A\rho}{(\rho + r_0)}\int_{r_l-1}^{r_l}\cos[\omega t - k(\rho + r)]dr. \tag{9}$$

Performing the integration,

$$E_l = \frac{-K_l(\theta)E_A\rho\lambda}{(\rho + r_0)} \sin[\omega t - k\rho - kr]_{r=r_{l-1}}^{r=r_l}. \quad (10)$$

and since $r_{l-1} = r_0 + (l-1)\lambda/2$ and $r_l = l\lambda/2$, Equation 10 can be evaluated, leading to

$$E_l = (-1)^{l+1} \frac{2K_l(\theta)E_A\rho\lambda}{(\rho + r_0)} \sin[\omega t - k(\rho + r_0)]. \quad (11)$$

The distance between adjacent zones differs by $\lambda/2$. Therefore, according to Equation 11, depending on whether l is even or odd, the energy term will be positive or negative respectively. As a result, the energy reaching the receiver from adjacent zones will be out of phase by half a wavelength and thus cancel each other. The total energy E reaching the receiver can be determined by accumulating the energy from each of the m zones

$$E = E_1 + E_2 + E_3 + \cdots + E_m \quad (12)$$

Since the sign of each zone alternates, Equation 12 can be re-formulated as

$$E = |E_1| - |E_2| + |E_3| - \cdots \pm |E_m| \quad (13)$$

From Equation 13, it can be deduced that the disturbance generated by the entire unobstructed wavefront is approximately equal to one half of the contribution of the first zone [10],

$$E \approx \frac{|E_1|}{2}. \quad (14)$$

IV. ACOUSTICAL DIFFRACTION USING THE HUYGENS-FRESNEL PRINCIPLE

The acoustical diffraction technique to be described here utilizes the Huygens-Fresnel Principle as described above to estimate the acoustical energy reaching a receiver from a given sound source after being diffracted by an edge. Essentially, given a sound source, receiver and edge, the energy reaching the receiver is estimated by considering the energy arriving at the receiver from the first Fresnel zone as in the unoccluded scenario. To account for diffraction effects, a visibility factor for the first Fresnel zone is introduced. The visibility factor (denoted by v_1) represents the fraction of the first zone visible from the receiver. Positions on the first zone are sampled uniformly and ray casting is used to determine the fraction of the zone visible to the receiver. The total visibility of the zone is equal to the fraction of sampled positions where a clear path between the sampled position and the receiver exists (n_{vis}), versus the total number of positions sampled (N_{vis}), given mathematically as $v_1 = n_{vis}/N_{vis}$

A. Method Details

In order to determine the energy arriving at the receiver from the i^{th} Fresnel zone (including the first zone), the position of one of the secondary sources within the i^{th} zone is required so that the obliquity factor ($K(\theta)$) and the sampled positions required to determine the visibility of the zone can be determined. In the sonel mapping method, at each sound source, sonels are emitted and traced through the environment while recording their interaction with any surfaces/objects they may encounter. Upon encountering a surface, a decision is made as to whether the sonel will be reflected specularly or diffusely, diffracted or completely absorbed (the decision is made probabilistically based on various parameters including frequency, distance to an edge etc.). If the sonel is to be diffracted, its position will be assumed to be on the edge (p_{edge}). Since the position of both the sound source and p_{edge} are known, the distance between them r_{se} can be determined. The radius of the primary wavefront is then set to this distance (e.g., $\rho = r_{se}$). Being on the edge itself, p_{edge} will be located on the surface of the wavefront and is assumed to be the position of one of the secondary sources in this particular Fresnel zone (Z_{init}). Although only the first zone is of interest, given the position of a secondary source in any other zone, referring to the geometry illustrated in Figures 2 and 3, the position of a secondary source in any other zone can be easily determined.

The initial Fresnel zone can be determined as

$$Z_{init} = \left\lfloor \frac{r_{init} - r_0}{\lambda} + 0.5 \right\rfloor \quad (15)$$

where, r_{init} is the distance between the receiver and p_{edge} and $r_0 = r_{sr} - \rho$ where, r_{sr} is the distance between the sound source and the receiver. Although p_{edge} may lie anywhere within Z_{init} and not necessarily on its boundary, it is assumed that the obliquity factor is constant throughout the entire zone.

The total energy reaching the receiver from the first zone Z_1 can be determined as follows:

$$\begin{aligned} E_1 &\approx v_1 \times \frac{|E_1|}{2} \\ &\approx v_1 \times |(-1)^2 \frac{2K_1(\theta)E_A\rho\lambda}{(\rho + r_0)} \sin[\omega t - k(\rho + r_0)]| \\ &\approx v_1 \times \frac{2K_1(\theta)E_A\rho\lambda}{(\rho + r_0)} \sin[\omega t - k(\rho + r_0)] \end{aligned} \quad (16)$$

where, $t = (r_0 + \lambda/2)/v_s$ is the time taken for the secondary sources on zone one to reach the receiver.

V. EXPERIMENTAL RESULTS

Two experiments are reported that indicate the effectiveness of the sonel acoustical diffraction method.

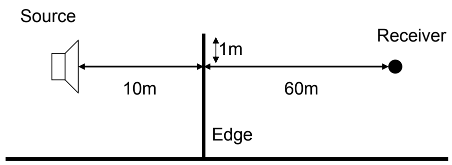

Fig. 4. Experiment one set-up.

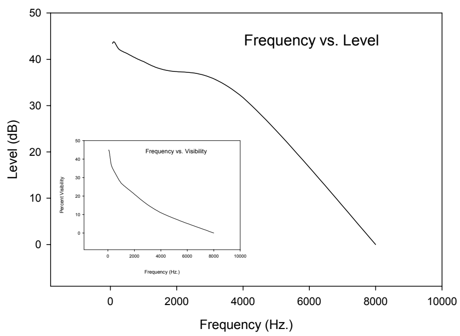

Fig. 5. Experiment one results. Frequency vs. sound level.

A. *Experiment one*

In this first experiment, the visibility of the first Fresnel zone was determined for various sound source frequencies given the sound source, receiver and the "infinite edge" configuration shown in Figure 4. The sound source and receiver were located at the same height and in line with each other and separated by an "infinite edge". The position of the single sonel on the edge and hence the position of a secondary sound source (when considering the Huygens-Fresnel approach) was also assumed to lie on the same (imaginary) line as the sound source and receiver. Frequencies examined were 63Hz, 125Hz, 250Hz, 500Hz, 1000Hz, 2000Hz, 4000Hz and 8000Hz.

A graphical summary of the results is provided in Figure 5 where the horizontal axis represents frequency and the vertical axis represents the level (in dB) of the sound reaching the receiver (the original sound source level was 90dB). The subplot included in the bottom left corner of Figure 5 describes frequency (horizontal axis) vs. visibility (vertical axis) of the first Fresnel zone from the receiver position. As seen in Figure 5, the visibility of zone 1 is inversely proportional to frequency. The decrease in visibility is due to a decrease in the size of the first Fresnel zone as frequency is increased. A decrease in visibility also leads to a decrease in the level of the sound reaching the receiver. As a result, as frequency increases, the sound level reaching the receiver decreases. This conforms to theoretical results that predict lower frequency sounds (and therefore longer associated wavelengths), are diffracted more [1].

B. *Experiment Two*

In the second experiment, a "non-infinite" edge plane (surface) with dimensions 6m $\times$ 6m was placed between the sound source and receiver. The configuration of the sound source,

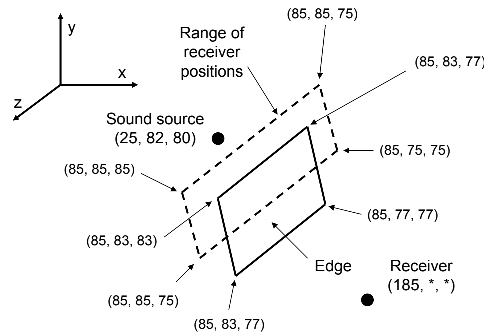

Fig. 6. Experiment two set-up.

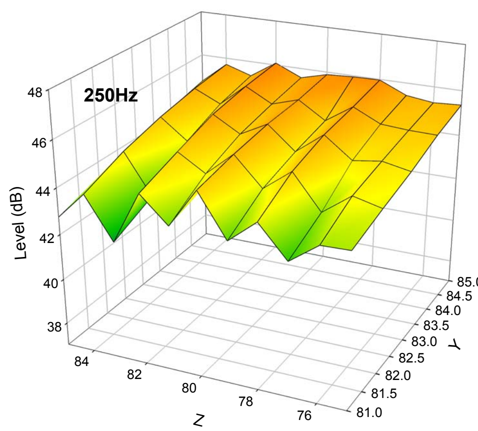

Fig. 7. Experiment two results: 250Hz.

edge plane and receiver is illustrated in Figure 6. The position of the sound source remained stationary while the position of the receiver was varied in one meter increments across the "Y" and "Z" coordinates, beginning at position (85, 75, 75) and ending at position (85, 85, 85). The sound source was positioned one meter below the edge (on the Y axis) and the receiver was positioned such that the "Y" and "Z" coordinates corresponded to the upper half of the edge. In this experiment, only edge effects were considered (e.g., no specular or diffuse reflections etc. were considered). For each receiver position, the sound level arriving at the receiver was determined by using the method described in the previous section.

The original sound level was 90dB and was equally divided amongst sonels that were emitted from the sound source. All of the sonels were diffracted on one the four edges of the plane. The actual edge was randomly chosen as was the position on the edge itself. Figure 7 and 8 illustrate the results for a sound source of 250Hz and 500Hz respectively. As indicated in both figures, the level increases as the position of the receiver increases from its initial value of Y = 80. In addition, as with the results of the previous experiment, a decrease in the sound level reaching the receiver is generally observed as frequency is increased.

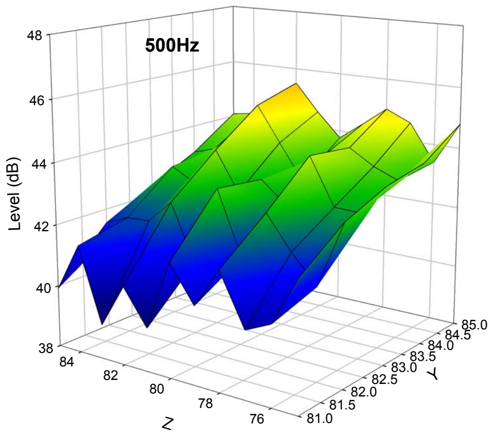

Fig. 8. Experiment two results: 500Hz.

VI. SUMMARY

This paper presented an acoustical diffraction modeling algorithm based on an approximation to the Huygens-Fresnel Principle. Preliminary results based on modeling of two simple configurations indicate the method can quickly and efficiently approximate acoustical diffraction effects. This work is ongoing and currently, the proposed diffraction method is being incorporated into the probabilistic (Russian Roulette) framework of the sonel mapping method. At each sonel-surface interaction point, based on several parameters of both the surface (e.g., size, diffuse and absorption coefficients) and the sound itself (e.g., frequency) a decision is made as to whether the sonel will be reflected specularly or diffusely, refracted, diffracted of absorbed. With the inclusion of the proposed diffraction modeling algorithm, sonel mapping is capable of modeling the acoustics of an environment, taking into account the various acoustical phenomena occurring as a sound propagates and interacts with objects/surfaces in the environment, in an efficient manner ultimately allowing it to be used in interactive virtual environments.

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