

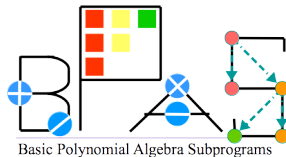
Multithreaded Programming in the BPAS Library

Building Object-Oriented (and Optional) Parallelization from C++ Primitives

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Introduction

```
1 void mergeSort(int* A, int i, int j) {  
2     if (j <= i) {  
3         return;  
4     }  
5     int k = i + (j-1)/2;  
6     mergeSort(A, i, k);  
7     mergeSort(A, k, j);  
8     merge(A, i, k, j);  
9 }
```

- Where recursive calls are independent, those function calls can be executed **concurrently**
- **Fork** the execution control flow and then **join** or **sync** them
- Hardware must support parallelism with multi-core or multi-processors

Compiler-Level Automatic Parallelization

- CILK and OPENMP provide automatic parallelization through compiler extensions
- Very easy but flexibility more challenging

```
void mergeSort(int* A, int i, int j) {  
    //... base case, k  
    cilk_spawn mergeSort(A, i, k);  
    mergeSort(A, k, j);  
    cilk_sync  
    merge(A, i, k, j);  
}
```

```
void mergeSort(int* A, int i, int j) {  
    //... base case, k  
    #pragma omp parallel num_threads(2)  
    {  
        #pragma omp sections {  
            #pragma omp section {  
                mergeSort(A, i, k);  
            }  
            #pragma omp section {  
                mergeSort(A, k, j);  
            }  
        }  
    }  
    merge(A, i, k, j);  
}
```

Dynamic Multithreading with BPAS

- Object-oriented, Standard C++11, No compiler extensions
- Supports composition of parallel regions
- **Dynamic Multithreading**: programmers specify opportunities for concurrency, runtime decides parallel execution

```
1 void mergeSort(int* A, int i, int j) {  
2     //... base case, k  
3     threadID id;  
4     ExecutorThreadPool& pool = ExecutorThreadPool::getThreadPool();  
5  
6     pool.obtainThread(id);  
7     pool.executeTask(id, std::bind(mergeSort, A, i, k));  
8     mergeSort(A, k, j);  
9     pool.waitForThread(id);  
10  
11     merge(A, i, k, j);  
12 }
```

Threading Primitives

C++11 introduced the *Thread Support Library*

■ `std::thread`

↳ C++ class encapsulating a thread (often a pthread) and its low-level spawn and join

■ `std::mutex`

↳ shared object between threads to indicate *mutual exclusion* to a **critical region**.

↳ mutex is *locked* or *owned* by at most one thread at a time.

■ `std::lock_guard`, `std::unique_lock`

↳ temporary object wrapping a mutex whose object lifetime automatically locks and unlocks the mutex.

↳ the constructor **blocks** and only returns once the shared mutex is successfully owned by the calling thread.

■ `std::condition_variable`

↳ blocks the current thread and temporarily releases a lock

↳ receives notification from another thread to awaken the blocked thread

std::function

Functors, function objects, callable objects

- First-class objects which are callable using normal function syntax
- Are often constructed by passing function names, function pointers
- `std::bind` binds arguments to a function or function object, returning a function object which requires fewer arguments

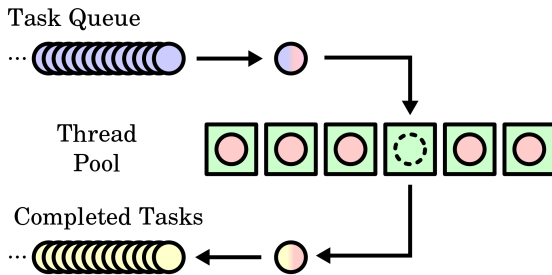
```
1 void printInteger(int a) {  
2     std::cout << a << std::endl;  
3 }  
4  
5 //Function object from function name  
6 std::function<void(int)> f_printInt(printInteger);  
7 f_printInt(12);  
8  
9 //Function object binding arguments to function name  
10 std::function<void()> f_print42 = std::bind(printInteger,42);  
11 f_print42();
```

Outline

- 1 Thread Pools
- 2 Parallel Patterns
- 3 Optional and Cooperative Parallelism

Thread Pools

- Highly parallel programs benefit for a thread pool: avoiding thread spawn overhead, **resource contention**
- Threads spawn only once at beginning of program
- Threads remain active throughout program lifetime



Function Executor Thread

FunctionExecutorThread

- Internal `std::thread` lifetime bound to object lifetime
- Executes `std::function` objects asynchronously
- `sendRequest(std::function<void()>)`: non-blocking submit task
- `waitForThread()`: blocking call until all tasks are complete
- Results available through object references or pointers

Function Executor Thread: Implementation

```
1  class FunctionExecutorThread {
2
3      AsyncObjectStream<std::function<void()>> requestQueue;
4      std::thread m_worker;
5
6      std::mutex m_mutex;
7      std::condition_variable m_cv;
8
9      FunctionExecutorThread() {
10         //member functions require pointer to member
11         m_worker = std::thread(
12             &FunctionExecutorThread::eventLoop, this);
13     }
14
15     void eventLoop();
16
17     void sendRequest(std::function<void()>);
18
19     void waitForThread();
20 }
```

Function Executor Thread: Implementation Details

```
1  void FunctionExecutorThread::eventLoop() {
2      std::function<void()> task;
3      while(requestQueue.getNextObject(task)) {
4          task();
5          std::unique_lock<std::mutex> lk(m_mutex);
6          bool notify = requestQueue.streamEmpty();
7          lk.unlock();
8          if (notify) m_cv.notify_all();
9      }
10 }
11
12 void FunctionExecutorThread::sendRequest(std::function<void()> f) {
13     std::lock_guard<std::mutex> lk(m_mutex);
14     requestQueue.addResult(f);
15 }
16
17 void FunctionExecutorThread::waitForThread() {
18     std::unique_lock<std::mutex> lk(m_mutex);
19     while (!requestQueue.streamEmpty()) {
20         m_cv.wait(lk);
21     }
22 }
```

Object Streams

The `AyncObjectStream` class provides:

- 1 a queue for tasks, or any object, and
 - 2 a blocking mechanism to keep the `FunctionExecutorThread` alive and idle when waiting for tasks
- Actually a class template for any kind of object being passed between two threads
 - Implements a queue satisfying the **producer-consumer problem**
 - A `std::queue` combined with a mutex and condition variable

AsyncObjectStream Interface

```
1  template <class Object>
2  class AsyncObjectStream {
3
4      std::queue<Object> retObjs;
5      std::mutex m_mutex;
6      std::condition_variable m_cv;
7      bool finished; //is the stream still open?
8
9      //Producer: add an object to the queue
10     void addResult(Object&& res);
11
12     //Producer: close the producer end of stream,
13     //           no more objects to produce
14     void resultsFinished();
15
16     //Consumer: pop an object from the queue, return true
17     //           iff stream is open and objects available
18     bool getNextObject(Object& res);
19
20     //Consumer: determine if queue is currently empty
21     bool streamEmpty();
22 };
```

AsyncObjectStream: getNextObject

```
1  bool getNextObject(Object& res) {
2      std::unique_lock<std::mutex> lk(m_mutex);
3      if (finished && retObjs.empty()) {
4          lk.unlock();
5          return false;
6      }
7
8      //Wait in a loop in case of spurious wake ups
9      while (!finished && retObjs.empty()) {
10         m_cv.wait(lk);
11     }
12
13     if (finished && retObjs.empty()) {
14         lk.unlock();
15         return false;
16     } else {
17         res = retObjs.front();
18         retObjs.pop();
19         lk.unlock();
20         return true;
21     }
22 }
```

ExecutorThreadPool

- A pool of **FunctionExecutorThreads**
- An internal queue of tasks and queue of threads
- When threads are busy, they are temporarily removed from the pool
- When all threads busy, tasks are added to task queue

```
1  class ExecutorThreadPool {
2
3  private:
4      std::deque<FunctionExecutorThread*> threadPool;
5      std::deque<std::function<void()>> taskPool;
6      std::mutex m_mutex;
7      std::condition_variable m_cv; // waitForThreads
8      std::vector<FunctionExecutorThread*> reservedThreads; // obtainThreads
9
10 public:
11     void addTask(std::function<void()> f);
12     void waitForThreads();
13 }
```

ExecutorThreadPool: Flexible Usage (1/2)

- In support of certain **parallel patterns**, clients can (temporarily) obtain exclusive ownership of threads from the pool, rather than using `addTask`
- Abstract away actual threads through **thread IDs**
- Once thread obtained, repeat Steps 2–3 as often as necessary

```
1  class ExecutorThreadPool {  
2  
3      //Step 1: obtain a thread's ID, removing it from the pool  
4      void obtainThread(threadID& id);  
5  
6      //Step 2: execute a task on a particular thread  
7      void executeTask(threadID id, std::function<void()>& f);  
8  
9      //Step 3 (optional): wait for thread to become idle  
10     void waitForThread(threadID id);  
11  
12     //Step 4: wait for thread and then return thread to pool  
13     void returnThread(threadID id);  
14 }
```

ExecutorThreadPool: Flexible Usage (2/2)

- In support of certain **parallel patterns**, clients can (temporarily) obtain exclusive ownership of threads from the pool, rather than using `addTask`
- Can obtain multiple threads at a time

```
1  class ExecutorThreadPool {
2
3      //Step 1: obtain threadIDs, removing them from the pool
4      void obtainThreads(std::vector<threadID>& ids);
5
6      //Step 2: execute a task on a particular thread
7      void executeTask(threadID id, std::function<void()>& f);
8
9      //Step 3 (optional): wait for threads to become idle
10     void waitForThreads(std::vector<threadID>& ids);
11
12     //Step 4: wait for threads and then return threads to pool
13     void returnThreads(std::vector<threadID>& ids);
14 }
```

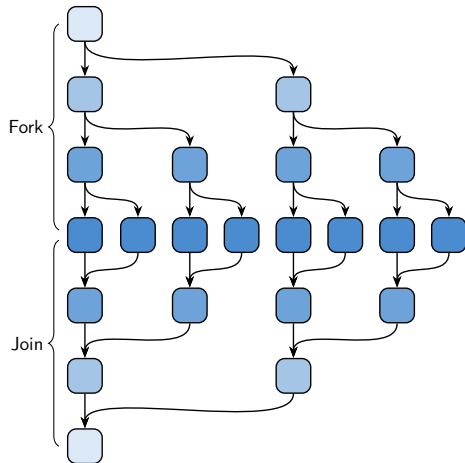
Outline

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Fork-Join



- **Fork**: divide problem and execute separate calls in parallel
- **Join**: merge parallel execution back into serial
- Recursively applying fork-join can easily parallelize a divide-and-conquer algorithm

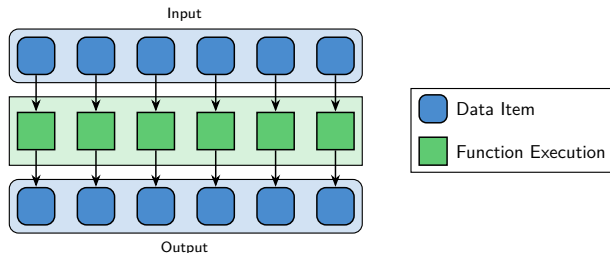
Fork-Join with ExecutorThreadPool

```
1 void mergeSort(int* A, int i, int j) {  
2     if (j <= i) { return; }  
3     int k = i + (j-1)/2;  
4     mergeSort(A, i, k);  
5     mergeSort(A, k, j);  
6     merge(A, i, k, j);  
7 }
```

```
1 void mergeSort(int* A, int i, int j) {  
2     if (j <= i) { return; }  
3     int k = i + (j-1)/2;  
4     threadID id;  
5     ExecutorThreadPool& pool = getThreadPool();  
6  
7     pool.obtainThread(id);  
8     pool.executeTask(id, std::bind(mergeSort, A, i, k));  
9     mergeSort(A, k, j);  
10  
11     pool.returnThread(id);  
12     merge(A, i, k, j);  
13 }
```

Map

- Simultaneously execute a function on each data item in a collection
- If more data items than threads, apply the pattern block-wise: partition the collection and apply one thread to each partition
- Often simplified as just a `parallel_for` loop
- Where multiple map steps are performed in a row, they should operate in lockstep



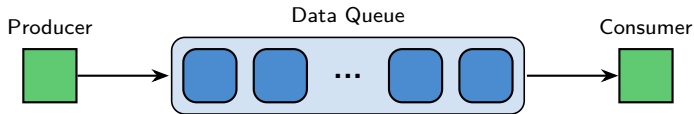
Map with ExecutorThreadPool

```
1 //Apply f to each item of A, returning results in B
2 void MapExample(int* B, int* A, int n, std::function<void(int*,int*)> f) {
3
4     for (int i = 0; i < n; ++i) {
5         f(&B[i], &A[i]);
6     }
7 }
```

```
1 void MapExample(int* B, int* A, int n, std::function<void(int*,int*)> f) {
2
3     ExecutorThreadPool& pool = getThreadPool();
4     std::vector<threadID> ids;
5     pool.obtainThreads(n-1, ids); //assume n-1 threads avail.
6
7     for (int i = 0; i < n-1; ++i) {
8         pool.executeTask(ids[i], std::bind(f, &B[i], &A[i]));
9     }
10    f(&B[n-1], &A[n-1]); //use main thread for one call
11
12    pool.returnThreads(ids); //also waits for threads
13 }
```

Producer-Consumer

- Two functions connected by a queue
- The producer produces data items, pushing them to the queue
- The consumer processes data items, pulling them from the queue
- Producer and consumer execute simultaneously; must avoid **deadlock**



- In some circumstances, the producer may be considered as an **iterator** or **generator**

Generators

- Generators are special kinds of **coroutines** which **yield** data items one at a time, rather than many as a collection
- A yield pauses execution of the function, and allows computations to resume from that point at the next function call

```
1 void FibonacciGen(int n) {  
2     int Fn_1 = 0;  
3     int Fn = 1;  
4     for (int i = 0; i < n; ++i) {  
5         yield Fn_1;  
6         Fn = Fn + Fn_1;  
7         Fn_1 = Fn - Fn_1;  
8     }  
9 }
```

- Where the generation of data items is itself expensive, generators may execute asynchronously, following the producer-consumer pattern: **AsyncGenerator**

AsyncGenerator Example

```
1 void FibonacciGen(int n, AsyncGenerator<int>& gen) {
2     int Fn_1 = 0;
3     int Fn = 1;
4     for (int i = 0; i < n; ++i) {
5         gen.generateObject(Fn_1); //yield Fn_1 and continue
6         Fn = Fn + Fn_1;
7         Fn_1 = Fn - Fn_1;
8     }
9     gen.setComplete();
10 }
11
12 void Fib() {
13     int n;
14     std::cin >> n;
15     AsyncGenerator<int> gen(FibonacciGen, n);
16
17     int fib;
18     //get one integer at a time until generator is finished
19     while (gen.getNextObject(fib)) {
20         std::cerr << fib << std::endl;
21     }
22 }
```

AsyncGenerator Implementation

```
1  template <class Object>
2  class AsyncGenerator {
3      AsyncObjectStream<Object> stream; //Objects instead of std::function
4      FunctionExecutorThread t;
5
6      //Create a generator from a function and its arguments
7      template <class Function, class... Args>
8      AsyncGenerator(Function& f, Args&...args) {
9
10         //pass reference to generator object to producer function
11         std::function<void()> boundF = std::bind(f, args..., std::ref(*this));
12         t.sendRequest(boundF);
13     }
14
15     //delegate to stream's methods
16     void generateObject(Object& obj) { stream.addResult(obj); }
17     void setComplete()               { stream.resultsFunished(); }
18     bool getNextObject(Object& obj)  { return stream.getNextObject(obj); }
19
20 }
```

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ExecutorThreadPool Optional Parallelism

- If all threads already reserved, `obtainThread(id)` may fail to find a thread
- In this case, return special value `const threadID notAThread`
- `executeTask(id, task)` behaves serially if `id == notAThread`
- Calls to `executeTask()` are a *suggestion* for parallelism, depending on the current state of the thread pool

```
1  class ExecutorThreadPool {  
2      void obtainThread(threadID& id);  
3  
4      void executeTask(threadID id, std::function<void()>& f);  
5  
6      void waitForThread(threadID id);  
7  
8      void returnThread(threadID id);  
9  }
```

ExecutorThreadPool Optional Parallelism (2/2)

```
1 void obtainThread(threadID& id) {
2     std::lock_guard<std::mutex> lk(m_mutex);
3     if (threadPool.empty()) {
4         id = ExecutorThreadPool::notAThread;
5     } else {
6         FunctionExecutorThread* t = threadPool.front();
7         threadPool.pop_front();
8         occupiedThreads.push_back(t);
9         id = t->get_id();
10    }
11 }
12
13 void executeTask(threadID id, std::function<void()>& f) {
14     if (id == ExecutorThreadPool::notAThread) {
15         f();
16     } else {
17         std::lock_guard<std::mutex> lk(m_mutex);
18         for (FunctionExecutorThread* t : occupiedThreads) {
19             if (t->get_id() == id) { t.sendRequest(f); break; }
20         }
21     }
22 }
```

AsyncGenerator Optional Parallelism

Can use thread pool rather than directly using a `FunctionExecutorThread`

→ If all threads in the pool are busy, execute the function serially instead

```
1  template <class Object>
2  class AsyncGenerator {
3
4      //Create a generator from a function and its arguments
5      template <class Function, class... Args>
6      AsyncGenerator(Function& f, Args&...args) {
7          auto boundF = std::bind(f, args..., std::ref(*this));
8
9          ExecutorThreadPool& pool = getThreadPool();
10         if (pool.allThreadsBusy()) {
11             boundF();
12         } else {
13             pool.addTask(f);
14         }
15     }
16 }
```

Cooperative Parallelism

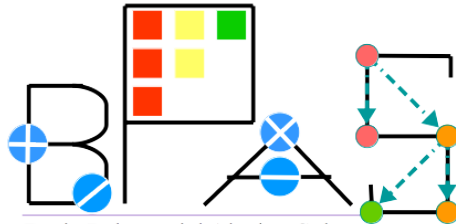
With simultaneous clients of `ExecutorThreadPool`, some tasks should be given **priority**.

- Some tasks are more **coarse-grained**, offer more potential speed-up
- Processing certain tasks may expose more parallelism and thus should be executed first

Solution: `pool.addPriorityTask()`

- If there are idle threads, priority task behaves as normal task
- If there are no idle threads:
 - 1 temporarily allow over-subscription and spawn a new **priority thread**
 - 2 assign the priority task to the new priority thread
 - 3 the next thread returned to the pool is **retired** to recover a state without over-subscription
- Do not spawn a priority thread if all current threads are priority threads

Thank you!



Basic Polynomial Algebra Subprograms

www.bpaslib.org

Questions?

ExecutorThreadPool Singleton

- To avoid *over-subscription*, a program should not use multiple thread pools
- All areas of code should share the same thread pool
- Use a classic singleton pattern

```
1  class ExecutorThreadPool {
2
3      private:
4          //pool size defaults to 1 less than hardware allows,
5          //the main thread counts as 1
6          ExecutorThreadPool(int nthreads =
7              std::thread::hardware_concurrency() - 1);
8
9      public:
10         static ExecutorThreadPool& getThreadPool() {
11             static ExecutorThreadPool pool;
12             return pool;
13         }
14     }
```

ExecutorThreadPool: addTask

```
1 void addTask(std::function<void()> f) {
2     std::unique_lock<std::mutex> lk(m_mutex);
3     taskPool.push_back(f);
4     lk.unlock();
5     tryPullTask();
6 }
7
8 void tryPullTask() {
9     std::unique_lock<std::mutex> lk(m_mutex);
10
11     if (!taskPool.empty() && !threadPool.empty()) {
12         FunctionExecutorThread* worker = threadPool.front();
13         threadPool.pop_front();
14
15         std::function<void()> f = taskPool.front();
16         taskPool.pop_front();
17         worker->sendRequest(f);
18     }
19
20     lk.unlock();
21 }
```

FunctionExecutorThread Callback (1/2)

How does a worker thread notify the thread pool that it has become idle?

→ A callback function inserts the thread itself back into the pool

```
1  ExecutorThreadPool(int nthreads) {
2      // for i = 0..nThreads
3      FunctionExecutorThread* t = new FunctionExecutorThread();
4      t->setCallback( std::bind(&ExecutorThreadPool::putBackThread, this) );
5  }
6
7  void putBackThread(FunctionExecutorThread* t) {
8      std::unique_lock<std::mutex> lk(m_mutex);
9      if (!taskPool.empty()) {
10         std::function<void()> f = taskPool.front();
11         taskPool.pop_front();
12         worker->sendRequest(f);
13     } else {
14         threadPool.push_back(t);
15     }
16     lk.unlock();
17     m_cv.notify_all(); //notify waitForThreads()
18 }
```

FunctionExecutorThread Callback (2/2)

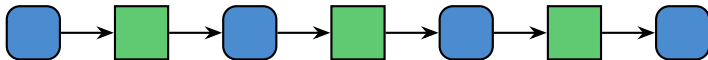
How does a worker thread notify the thread pool that it has become idle?

→ A callback function inserts the thread itself back into the pool

```
1  class FunctionExecutorThread {
2
3      std::function<void(FunctionExecutorThread*)> cb;
4
5      void eventLoop() {
6          std::function<void()> task;
7          while(requestQueue.getNextObject(task)) {
8              task();
9
10             if (cb) cb((FunctionExecutorThread*) this);
11
12             std::unique_lock<std::mutex> lk(m_mutex);
13             bool notify = requestQueue.streamEmpty();
14             lk.unlock();
15             if (notify) m_cv.notify_all();
16         }
17     }
18 }
```

Pipeline

- A sequence of stages where the output of one stage is used as the input to another
- Two consecutive stages form a producer-consumer pair
- Internal stages are both producer and consumer
- Typically, a pipeline is constructed statically through code organization
- Pipelines can be created dynamically and implicitly with AsyncGenerators and the callstack



Pipelines with AsyncGenerator

```
1 void intSequence(AsyncGenerator<int>& prevStage, AsyncGenerator<int>&
   nextStage) {
2     int i;
3     while(prevStage.getNextObject(i)) {
4         nextStage.generateObject(i);
5     }
6 }
7
8 std::vector<int> A = {1,2,3,4,5,6,7,8,9};
9 AsyncGenerator<int> stageOne(A);
10
11 AsyncGenerator<int> stageTwo(intSequence, stageOne);
12 AsyncGenerator<int> stageThree(intSequence, stageTwo);
13 AsyncGenerator<int> stageFour(intSequence, stageThree);
14
15 //consume from last stage of pipeline
16 int i;
17 while(stageFour.getNextObject()) {
18     std::cout << i << std::endl;
19 }
```