## Assignment 6 CSCI 3110: Design and Analysis of Algorithms

Due July 3, 2018

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Assignments are due on the due date before class and have to include this cover page. Plagiarism in assignment answers will not be tolerated. By submitting their answers to this assignment, the authors named above declare that its content is their original work and that they did not use any sources for its preparation other than the class notes, the textbook, and ones explicitly acknowledged in the answers. Any suspected act of plagiarism will be reported to the Faculty's Academic Integrity Officer and possibly to the Senate Discipline Committee. The penalty for academic dishonesty may range from failing the course to expulsion from the university, in accordance with Dalhousie University's regulations regarding academic integrity. Question 1 (10 marks) Use induction to prove that the following recurrences have the stated solutions.

- (a) T(n) = 3T(n/2) + n. Solution:  $T(n) \in \Theta(n^{\lg_2 3})$ .
- (b) T(n) = 3T(n/4) + T(n/5) + n. Solution:  $T(n) \in \Theta(n)$ .
- (c)  $T(n) = 2\sqrt{n}T(\sqrt{n}) + n$ . Solution:  $T(n) \in \Theta(n \lg n)$ .

Hint: For the upper bound, you want to prove that  $T(n) \le c_1 n \lg n - c_2 n$  for  $n \ge n_0$ .

Note that, in order to prove that  $T(n) \in \Theta(f(n))$ , you need to prove that  $T(n) \in O(f(n))$  and  $T(n) \in \Omega(f(n))$ .

**Question 2 (10 marks)** Solve each of the following recurrences using the Master Theorem. State which case applies and show that the recurrence satisfies the conditions of this case. It is also possible that the Master Theorem is not applicable to some of these recurrences. If the Master Theorem is not applicable, state that it isn't and explain why not.

- (a)  $T(n) = 4T(n/3) + n \lg n$
- (b)  $T(n) = 4T(n/2) + n^2/\lg n$
- (c)  $T(n) = 9T(n/3) + n^2$
- (d) T(n) = 3T(n/4) + n
- (e)  $T(n) = 2T(n/2) + n \lg n$ .