Assignment 5

CSCI 3110: Design and Analysis of Algorithms

Due June 19, 2018

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Assignments are due on the due date before class and have to include this cover page. Plagiarism in assignment answers will not be tolerated. By submitting their answers to this assignment, the authors named above declare that its content is their original work and that they did not use any sources for its preparation other than the class notes, the textbook, and ones explicitly acknowledged in the answers. Any suspected act of plagiarism will be reported to the Faculty's Academic Integrity Officer and possibly to the Senate Discipline Committee. The penalty for academic dishonesty may range from failing the course to expulsion from the university, in accordance with Dalhousie University's regulations regarding academic integrity.

Back to interval scheduling. In class, we discussed how to maximize the number of classes we can schedule without conflicts in a single classroom. Now, scheduling only some of the classes and just cancelling the remaining ones because we cannot schedule all classes in the same classroom isn't really an option. So here is a much more realistic formulation of the problem. You are given n classes, each represented by an interval defined by its starting time and its ending time. (Note, as in class, we are given actual start and ending times that cannot be moved, not just the duration of each class.) As in class, two classes can be scheduled in the same classroom only if their time intervals do not overlap. Your goal is to schedule *all* classes and to minimize the number of classrooms needed to accommodate them all.

(a) Provide a simple greedy algorithm that solves this problem in $O(n \lg n + kn)$ time, where k is the number of classrooms the schedule you compute requires.

(b) Prove that your algorithm produces a valid schedule that uses the minimum number of classrooms: no two classes scheduled in the same room by your algorithm overlap and, if your schedule uses k classrooms, there is no way to accommodate all the given classes in less than k classrooms. Your proof should use the fact that, if there is a time t that is contained in the time intervals of k classes, then each of these classes needs to be scheduled in a different classroom, that is, there cannot be any schedule that uses less than k classrooms.

(c) Argue briefly that the running time of your algorithm is indeed $O(n \lg n + kn)$.

(d) Optional (25% bonus marks): Discuss how to modify the implementation details, not the overall strategy, of your algorithm so its running time is reduced to $O(n \lg n + n \lg k) = O(n \lg n)$. Argue briefly that this modification indeed achieves this running time.

Hint: The reason why Serikzhan discussed decomposing a sequence of numbers into a minimum number of monotonically increasing subsequences in the last tutorial is because the problem in this assignment is very similar to it and in fact a bit simpler:

- The algorithm for decomposing into monotonically increasing subsequences started with a single sequence and only added a new sequence every time it was impossible to add the next number to any of the existing sequences. A similar strategy should work here; the key in this case is sorting the intervals in the right order so you can guarantee that, whenever you add a new classroom to accommodate the next class, the set of classes seen so far indeed cannot be scheduled in fewer classrooms.

- The correctness proof of the algorithm for decomposing into monotonically increasing subsequences used the fact that, if there's a monotonically decreasing subsequence of length k, then the decomposition must have at least k increasing subsequences. The argument in point (b) above is similar: if you need to run k classes in parallel (that is, there is a time when all of these k classes are simultaneously in progress), then you need at least k classrooms.