Fundamentals of Computational Neuroscience 2e

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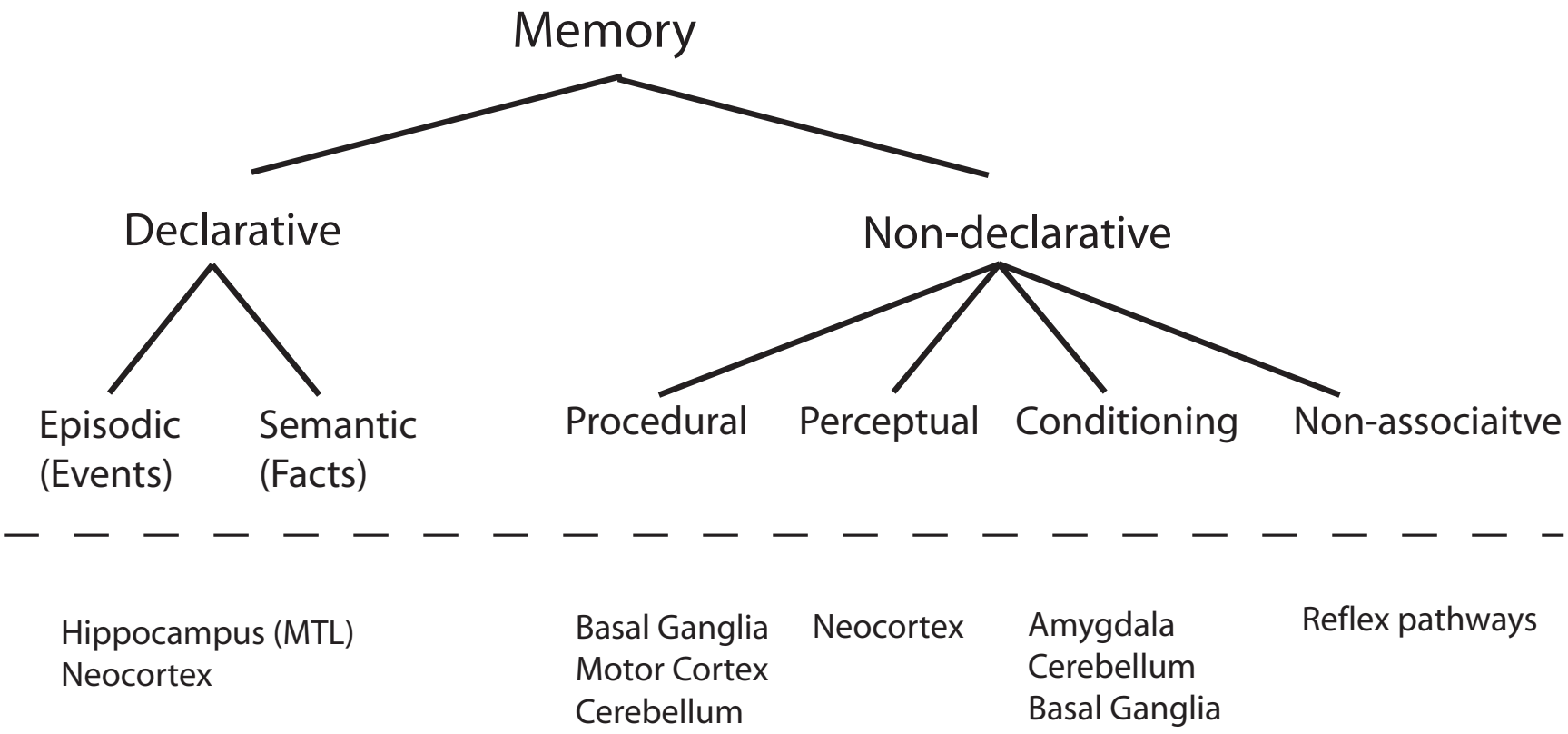
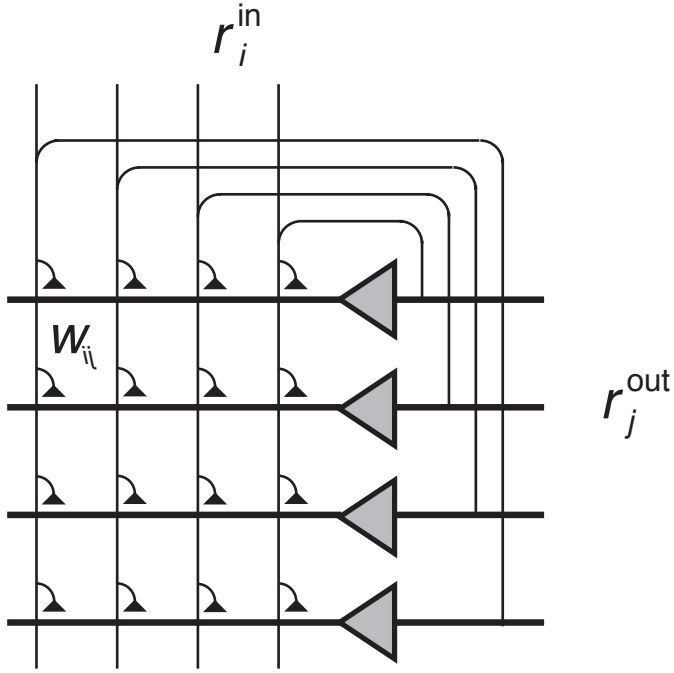
Chapter 8: Recurrent associative networks and episodic memory

Memory classification scheme (Squire)

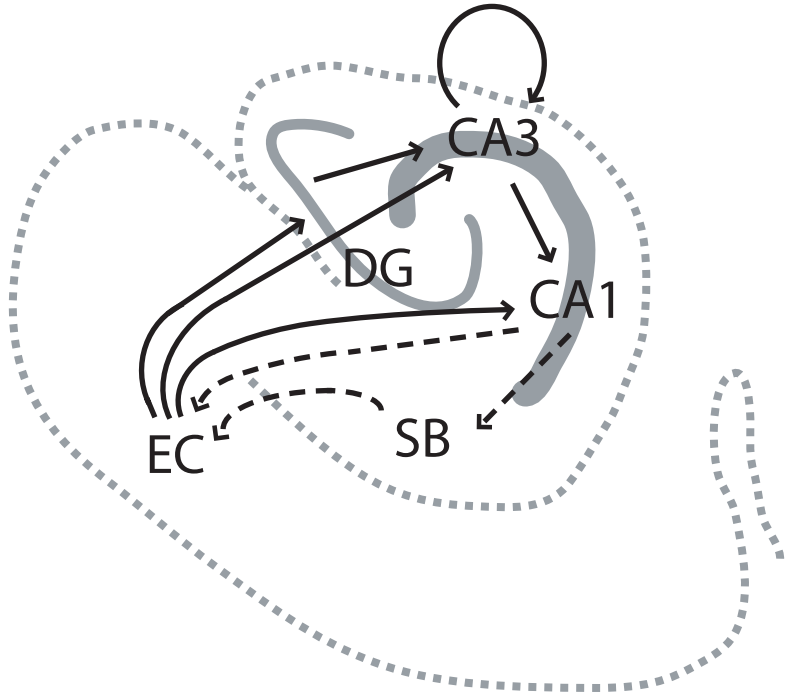


Auto-associative network and hippocampus

A. Recurrent associator network



B. Schematic diagram of the Hippocampus



Point attractor neural network (ANN)

Update rule: $\tau \frac{\mathrm{d}u_i(t)}{\mathrm{d}t} = -u_i(t) + \frac{1}{N}\sum_j w_{ij} r_j(t) + I_i^{\mathrm{ext}}(t)$

Activation function $r_i = g(u_i)$ (e.g. threshold functions)

Learning rule $w_{ij} = \epsilon \sum_{\mu=1}^{N_p} (r_i^\mu - \langle r_i \rangle)(r_j^\mu - \langle r_j \rangle) - c_i$

Training patterns: Random binary states with components $s_i^\mu \in \{-1, 1\}$, $r_i = \frac{1}{2}(s_i + 1)$

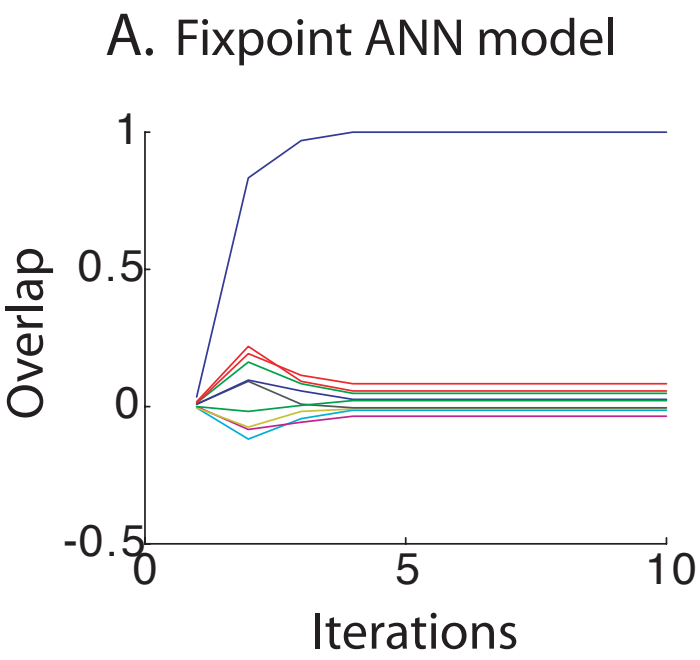
Update equations for fixed-point model($\mathrm{d}u_i/\mathrm{d}t = 0$)**:**
$s_i(t+1) = \mathrm{sign}\left(\sum_j w_{ij} s_j(t)\right)$

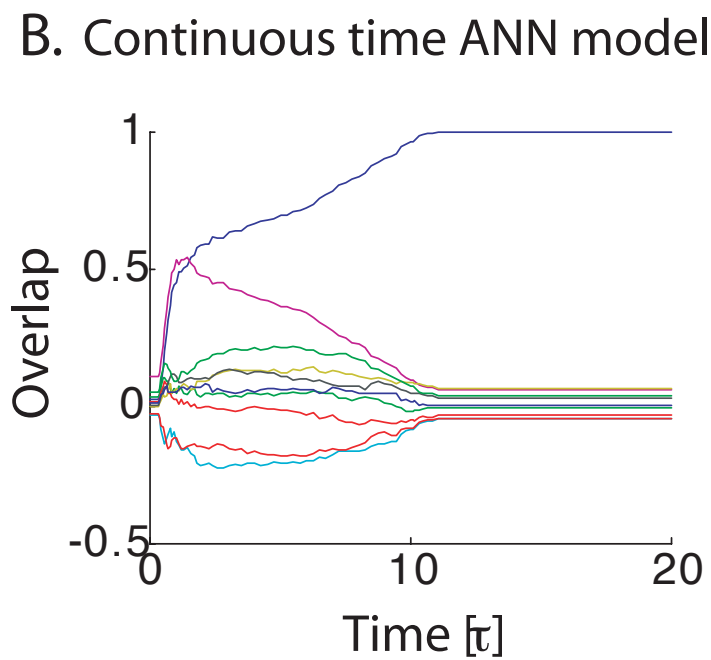
ann_cont.m

```
1   %% Continuous time ANN
2    clear; clf; hold on;
3    nn = 500; dx=1/nn; C=0;
4
5   %% Training weight matrix
6    pat=floor(2*rand(nn,10))-0.5;
7    w=pat*pat'; w=w/w(1,1); w=100*(w-C);
8   %% Update with localised input
9    tall = []; rall = [];
10   I_ext=pat(:,1)+0.5; I_ext(1:10)=1-I_ext(1:10);
11   [t,u]=ode45('rnn_ode_u',[0 10],zeros(1,nn),[],nn,dx,w,I_ext);
12   r=u>0.; tall=[tall;t]; rall=[rall;r];
13  %% Update without input
14   I_ext=zeros(nn,1);
15   [t,u]=ode45('rnn_ode_u',[10 20],u(size(u,1),:),[],nn,dx,w,I_ext);
16   r=u>0.; tall=[tall;t]; rall=[rall;r];
17  %% Plotting results
18   plot(tall,4*(rall-0.5)*pat/nn)
```

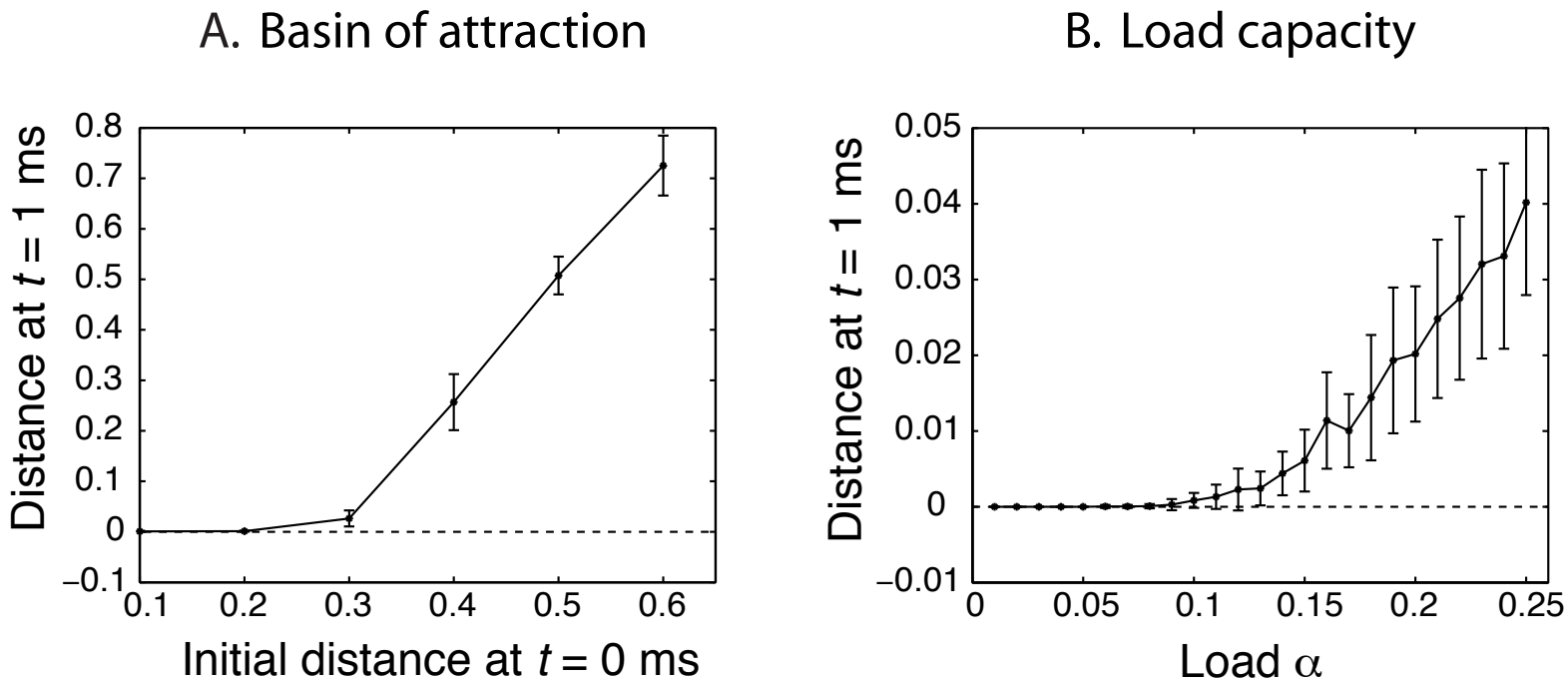
ann_fixpoint.m

```
1   pat=2*floor(2*rand(500,10))-1;          % Random binary pattern
2   w=pat*pat';                             % Hebbian learning
3   s=rand(500,1)-0.5;                      % Initialize network
4   for t=2:10; s(:,t)=sign(w*s(:,t-1)); end   % Update network
5   plot(s'*pat/500)
```





Memory breakdown



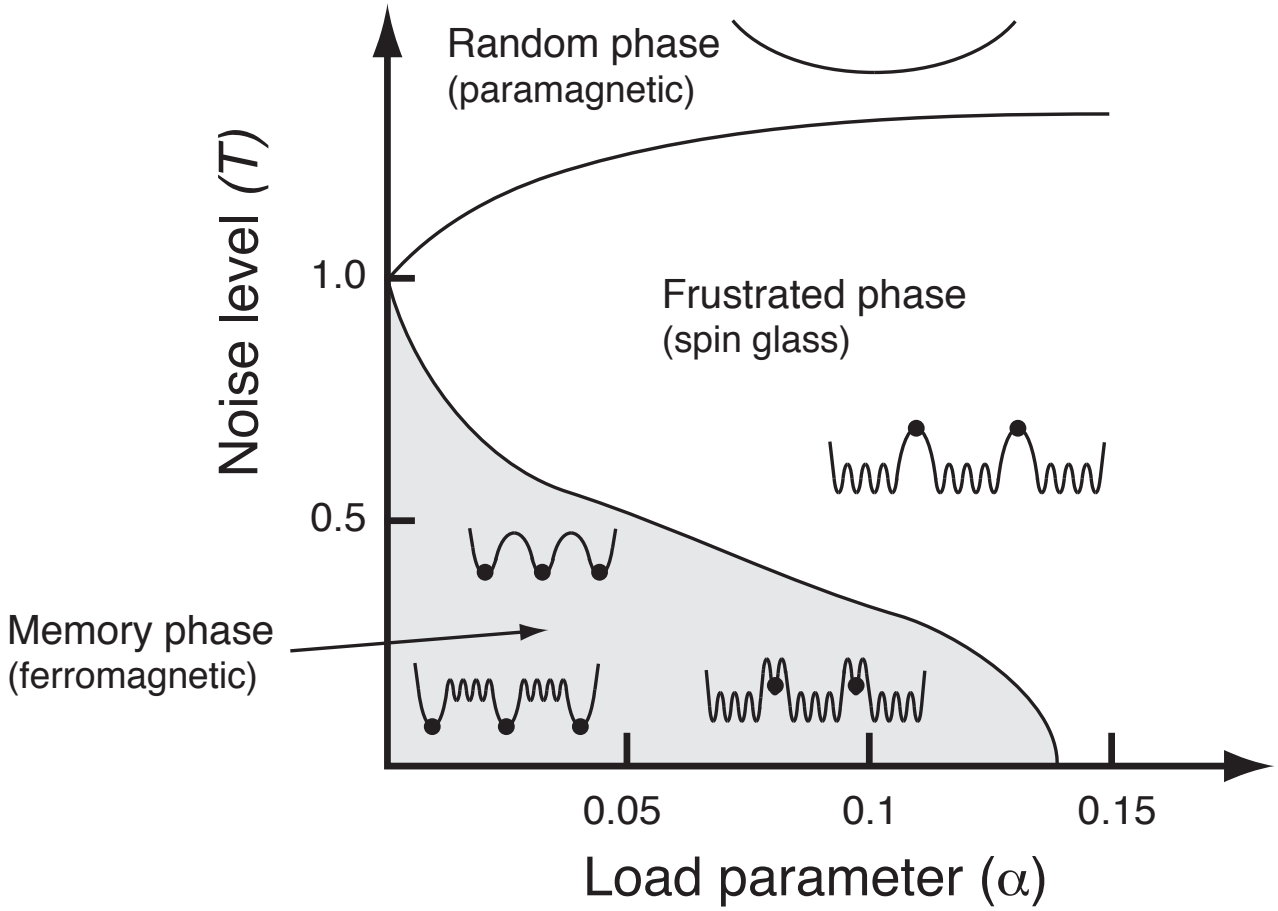
Probabilistic update rule:

$$P(s_i(t) = \pm 1) = \frac{1}{1 + \exp(\mp 2 \sum_j w_{ij} s_j(t-1)/T)}$$

Recovers deterministic rule in $\lim_{T \to 0}$

$$s_i(t) = \text{sign}(\sum_j w_{ij} s_j(t-1))$$

Phase diagram



How to minimize interference between patterns?

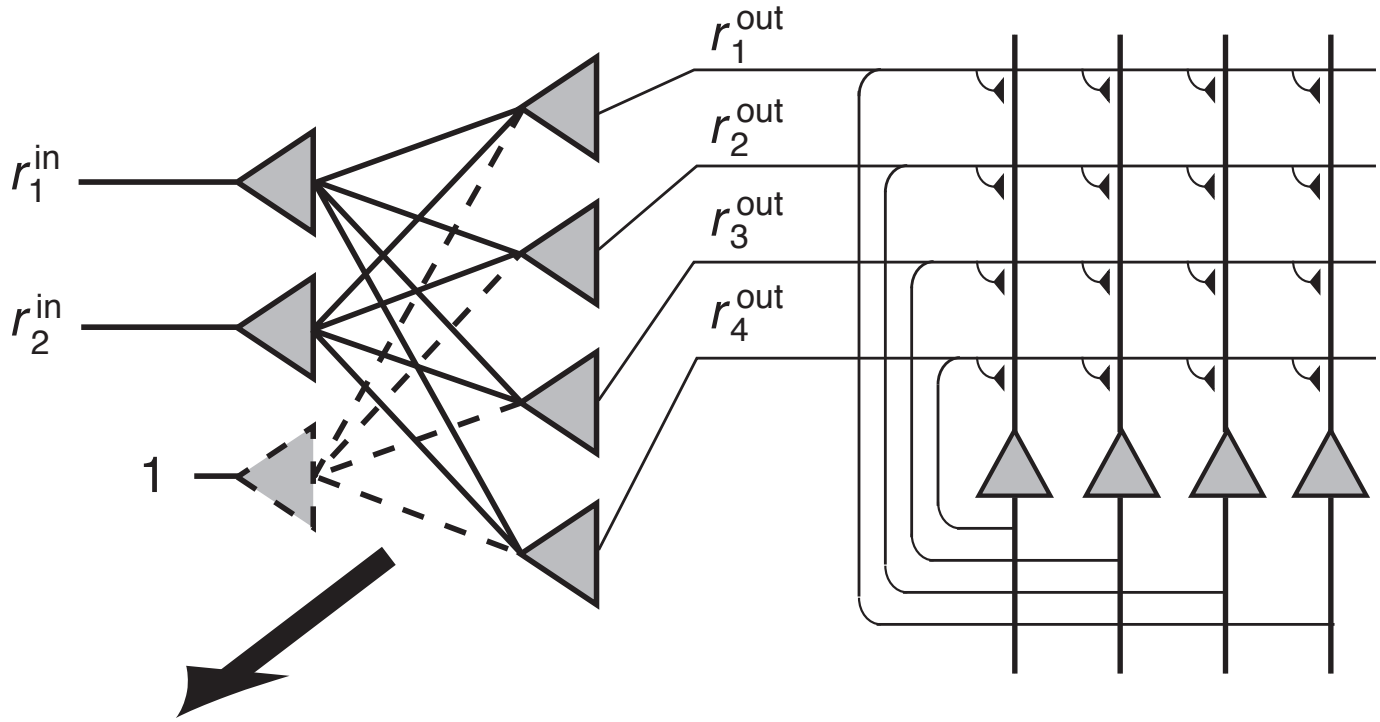
Associative memory in ANN is strongly influenced by interference between patter due to

- correlated patterns
- random overlap

Storage capacity can be much enhanced through decorrelating patterns. Simplest approach is generating sparse representations with expansion re-coding.

Storage capacity: $\alpha_c \approx \frac{k}{a \ln(1/a)}$ (Rolls & Treves)

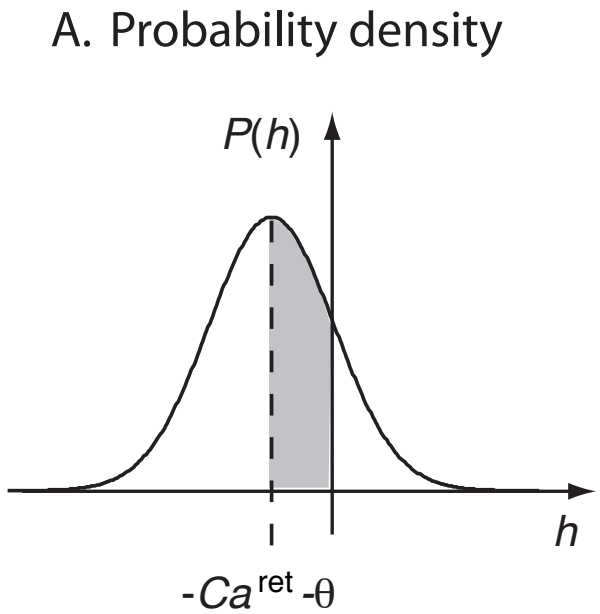
Expansion re-coding (e.g. in dentate gyrus)

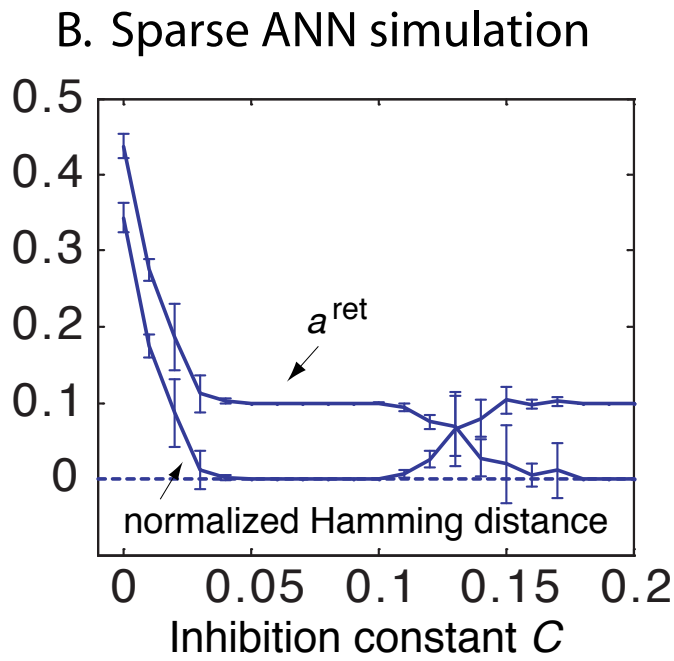


$$\begin{pmatrix} -1 & -1 & 0.5 \\ 1 & -1 & -0.5 \\ -1 & 1 & -0.5 \\ 1 & 1 & -1.5 \end{pmatrix}$$

r_1^{in}	r_2^{in}	r_1^{out}	r_2^{out}	r_3^{out}	r_4^{out}
0	0	1	0	0	0
1	0	0	1	0	0
0	1	0	0	1	0
1	1	0	0	0	1

Sparse pattern and inhibition



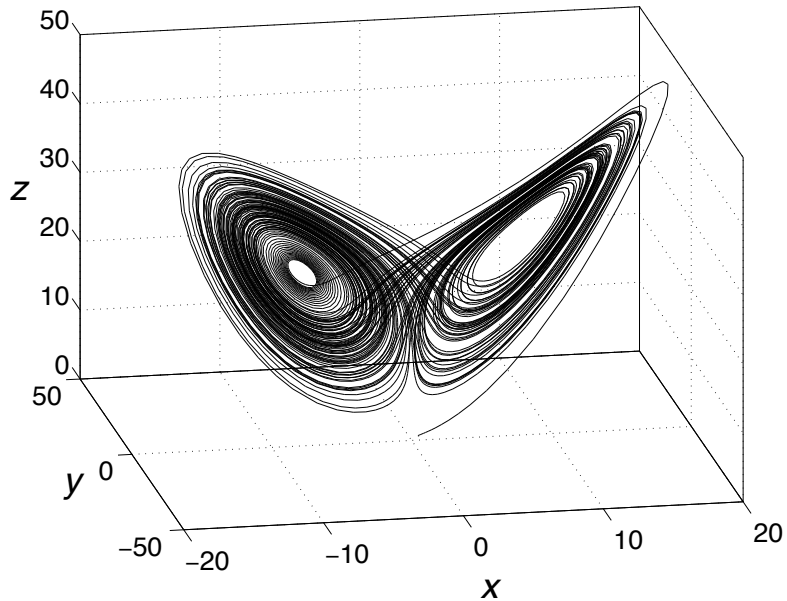


More general dynamical systems

Example: 3 nodes with $\sum$ and $\sum - \prod$ coupling (Lorenz attractor):

$$\frac{\mathrm{d}x_i}{\mathrm{d}t} = \sum_j w^1_{ij} x_i + \sum_{jk} w^2_{ijk} x_j x_k$$

$$\mathbf{w}^1 = \begin{pmatrix} -1 & a & 0 \\ b & -1 & 0 \\ 0 & 0 & -c \end{pmatrix} \quad \text{and} \quad \mathbf{w}^2 = \begin{cases} w^2_{213} = -1 \\ w^2_{312} = -1 \\ 0 \quad \text{otherwise} \end{cases}$$



Cohen–Grossberg theorem

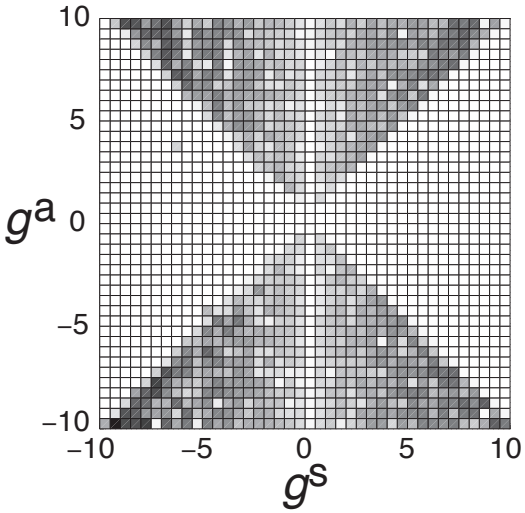
Dynamical system of the form $\frac{dx_i}{dt} = -a_i(x_i)\left(b_i(x_i) - \sum_{j=1}^{N}(w_{ij}g_j(x_j))\right)$

Has a Lyapunov (Energy) function, which guaranties point attractors, under the conditions that

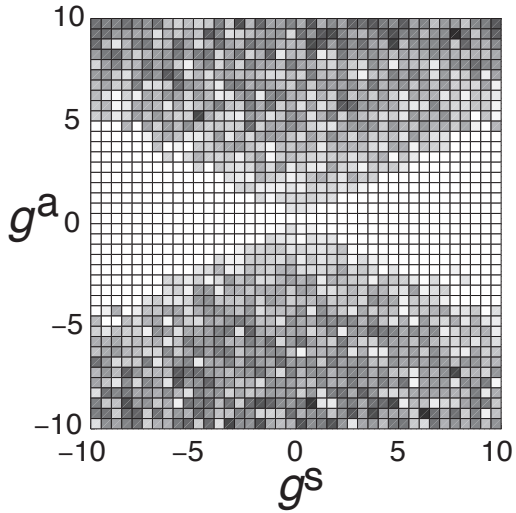
1. **Positivity** $a_i \geq 0$**:** The dynamics must be a leaky integrator rather than an amplifying integrator.
2. **Symmetry** $w_{ij} = w_{ji}$**:** The influence of one node on another has to be the same as the reverse influence.
3. **Monotonicity** $\operatorname{sign}(\mathrm{d}g(x)/\mathrm{d}x) = \mathrm{const}$**:** The activation function has to be a monotonic function.

Recurrent networks with non-symmetric weights

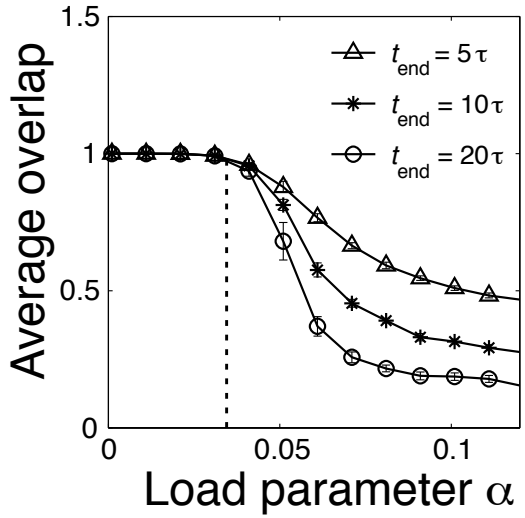
A. Unit components



B. Random components



C. Hebb-Dale network



More general dynamics

Non-monotone activation functions (tuning curves)

Networks with hidden nodes

Further Readings

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