Fundamentals of Computational Neuroscience 2e

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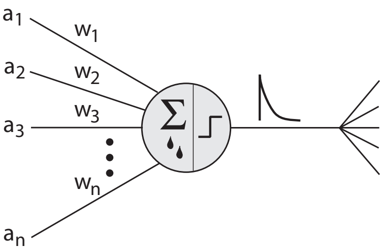
Chapter 3: Simplified neuron and population models

The leaky integrate-and-fire neuron

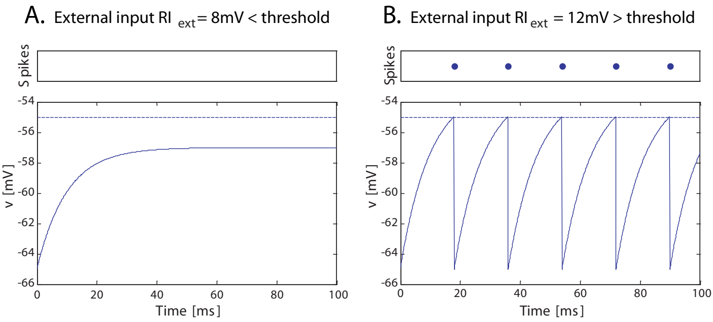
$$\tau_m \frac{\mathrm{d}v(t)}{\mathrm{d}t} = -(v(t) - E_L) + RI(t), \tag{1}$$

$$v(t^f) = \vartheta. \tag{2}$$

$$\lim_{\delta \to 0} v(t^f + \delta) = v_{res}, \tag{3}$$



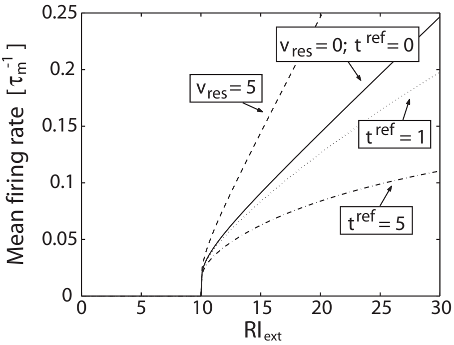
The leaky integrate-and-fire neuron (cont.)



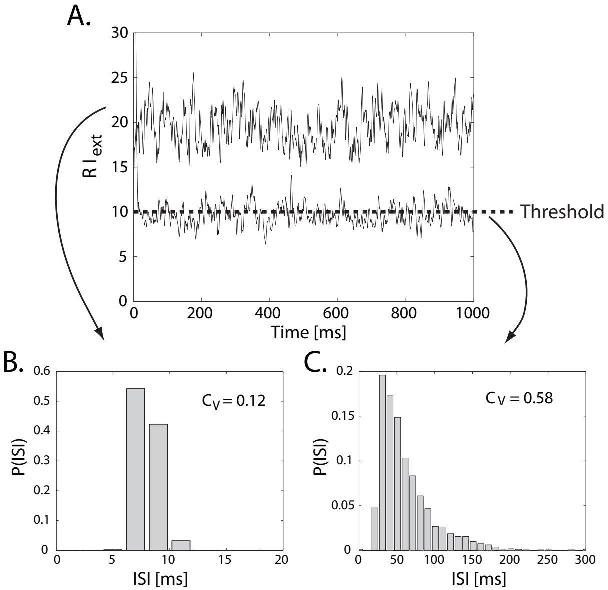
The LIF-neuron (cont.): Gain function

The inverse of the first passage time defines the **firing rate**

$$\bar{r} = (t^{\text{ref}} - \tau_{\text{m}} \ln \frac{\vartheta - RI}{v_{\text{res}} - RI})^{-1} \tag{4}$$



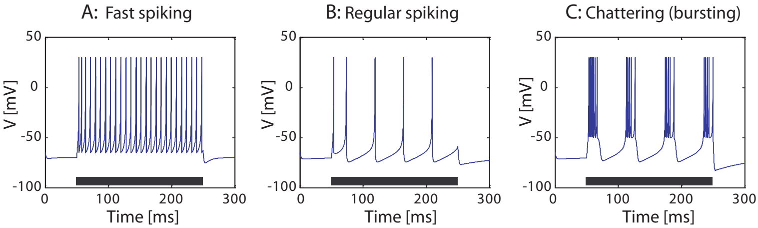
The LIF-neuron (cont.): Noise



The Izhikevich neuron

$$\frac{\mathrm{d}v(t)}{\mathrm{d}t} = 0.04v^2(t) + 5v(t) + 140 - u + I(t)$$
$$\frac{\mathrm{d}u(t)}{\mathrm{d}t} = a(bv - u)$$

$$v(v > 30) = c \;\text{ and }\; u(v > 30) = u - d$$

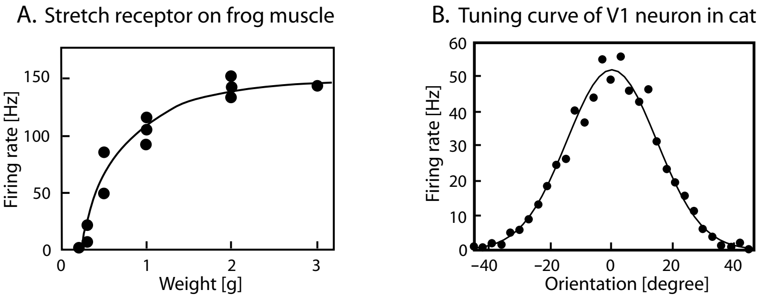


McCulloch-Pitts neuron

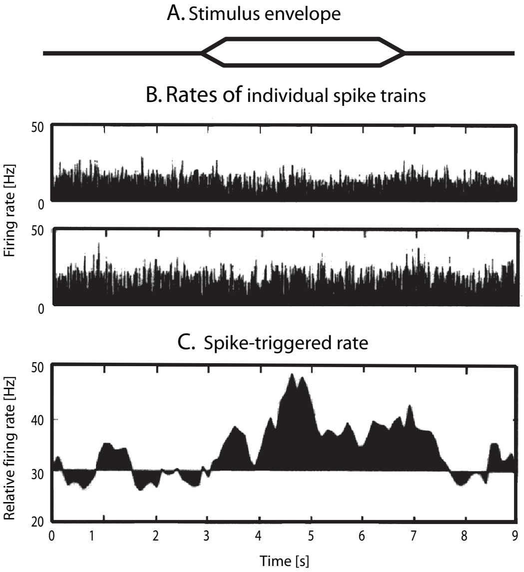
$$h = \sum_i x_i^{\mathrm{in}}$$

$$x^{\mathrm{out}} = \begin{cases} 1 & \text{if } h > \Theta \\ 0 & \text{otherwise} \end{cases}$$

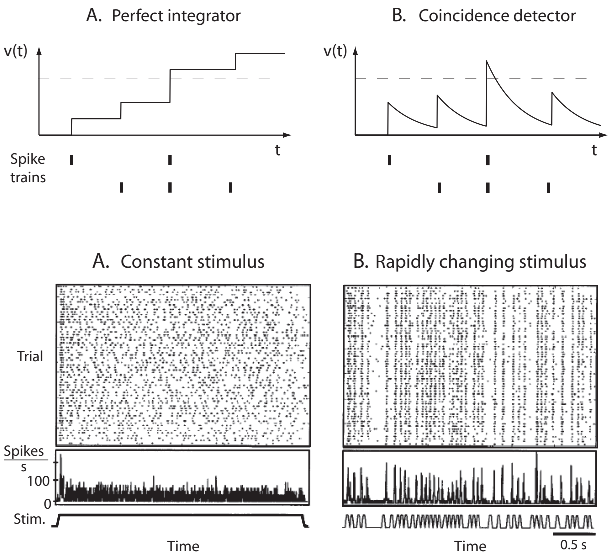
The firing rate hypothesis



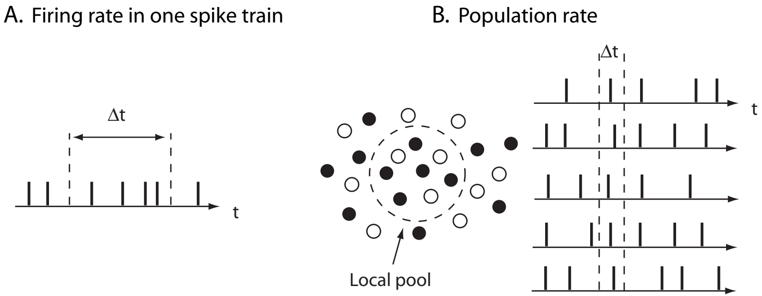
Counter example: correlation code (?)



Coincidence detector



Population model



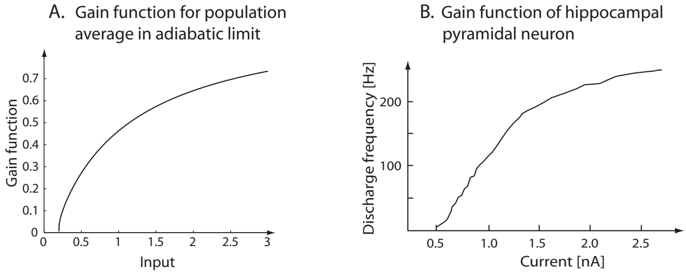
Population dynamics

For slow varying input (adiabatic limit), when all nodes do practically the same, same input, etc (Wilson and Cowan, 1972):

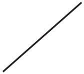
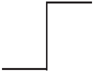
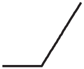
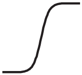
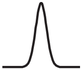
$$\tau \frac{\mathrm{d}A(t)}{\mathrm{d}t} = -A(t) + g(RI^{\mathrm{ext}}(t)). \tag{5}$$

Gain function:

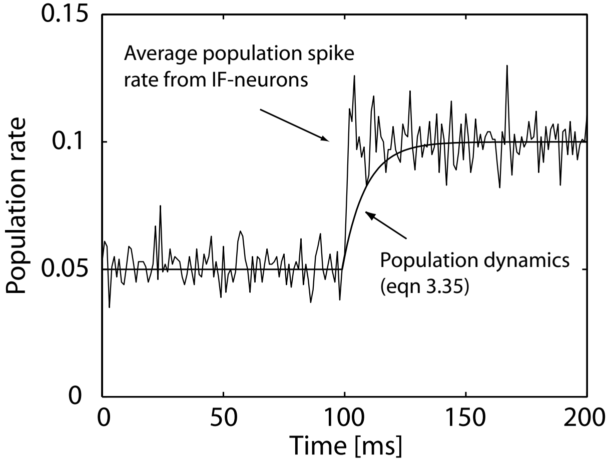
$$g(x) = \frac{1}{t^{\mathrm{ref}} - \tau \log(1 - \frac{1}{\tau x})}, \tag{6}$$



Other gain functions

Type of function	Graphical represent.	Mathematical formula	Matlab implementation
Linear		$g^{lin}(x) = x$	`X`
Step		$g^{step}(x) = \begin{cases} 1 & \text{if } x > 0 \\ 0 & \text{elsewhere} \end{cases}$	`floor(0.5*(1+sign(x)))`
Threshold-linear		$g^{theta}(x) = x\,\Theta(x)$	`x.*floor(0.5*(1+sign(x)))`
Sigmoid		$g^{sig}(x) = \frac{1}{1+\exp(-x)}$	`1./(1+exp(-x))`
Radial-basis		$g^{gauss}(x) = \exp(-x^2)$	`exp(-x.^2)`

Fast population response!!!



Further Readings

Wolfgang Maass and Christopher M. Bishop (eds.) (1999), **Pulsed neural networks**, MIT Press.

Wulfram Gerstner (2000), **Population dynamics of spiking neurons: fast transients, asynchronous states, and locking**, in **Neural Computation** 12: 43–89.

Eugene M. Izhikevich (2003), **Simple Model of Spiking Neurons**, in **IEEE Transactions on Neural Networks**, 14: 1569–1072.

Eugene M. Izhikevich (2004), **Which model to use for cortical spiking neurons?**, in **IEEE Transactions on Neural Networks**, 15: 1063–1070.

Warren McCulloch and Walter Pitts (1943) **A logical calculus of the ideas immanent in nervous activity**, in**Bulletin of Mathematical Biophysics** 7:115–133.

Huge R. Wilson and Jack D. Cowan (1972), **Excitatory and inhibitory interactions in localized populations of model neurons**, in **Biophys. J.** 12:1–24.

Nicolas Brunel and Xiao-Jing Wang, (2001), **Effects of neuromodulation in a cortical network model of working memory dominated by recurrent inhibition**, in **Journal of Computational Neuroscience** 11: 63–85.