Question (30 marks) Apply the Pumping Lemma to prove that the following languages are not regular.

(a) All binary strings with at least as many 0s as 1s.

(b) All binary strings consisting exactly one 1, either in the first quarter of the string or in the last quarter of the string. For example, the strings 01000000 and 0001 belong to this language. The strings 0000 and 00001000 do not. For strings that have a length not divisible by 4, we say that the 1 belongs to the first quarter if its position divided by the length of the string is at most 1/4. Similarly, to belong to the last quarter, the position divided by the length of the string has to be strictly greater than 3/4. Thus, the string 001 also belongs to the language, while the string 01000 does not.

(c) All strings $\sigma$ over the alphabet \{\(,\)\} such that $\sigma)))$ would be a string of properly nested parentheses. For example, the strings ((( and (((((()()(((( belong to this language—three closing parentheses are needed to close the parentheses that are still open, and every closing parenthesis matches a preceding opening parenthesis—while the strings ((( and (((((()()))) do not—the former has 4 parentheses that are open and need to be closed, the latter has none.