Assignments are due on the due date before class and have to include this cover page. Plagiarism in assignment answers will not be tolerated. By submitting their answers to this assignment, the authors named above declare that its content is their original work and that they did not use any sources for its preparation other than the class notes, the textbook, and ones explicitly acknowledged in the answers. Any suspected act of plagiarism will be reported to the Faculty's Academic Integrity Officer and possibly to the Senate Discipline Committee. The penalty for academic dishonesty may range from failing the course to expulsion from the university, in accordance with Dalhousie University’s regulations regarding academic integrity.
Question 1 (10 marks) In class, I claimed that the worst-case number of iterations the proposal algorithm for the stable marriage problem takes to terminate is quadratic in the number of men and women. Describe a family of inputs with one input per input size $n$ so that the algorithm takes a quadratic number of iterations for each of these inputs. Prove that the algorithm takes a quadratic number of iterations for each of these inputs. For 8/10 marks, it is sufficient to prove a quadratic bound. For 10/10 marks, you should provide a family of inputs such that the input of size $n$ forces the algorithm to run for $n^2 - n + 1$ iterations. For 5 bonus marks, that is, 15/10 marks, either provide an input family that forces the algorithm to run for more than $n^2 - n + 1$ iterations or, in addition to providing an input family that forces $n^2 - n + 1$ iterations, prove that no input forcing more than $n^2 - n + 1$ iterations exists.

Question 2 (10 marks) A priority queue is a data structure that supports the following operations:

- An Insert($Q, x, p$) inserts an element $x$ into $Q$ with priority $p$. You may assume $x$ is not currently in $Q$.
- A Delete($Q, x$) operation deletes element $x$ from $Q$. You may assume $x$ is in $Q$.
- A DecreaseKey($Q, x, p$) assumes $x$ is currently in $Q$ and has priority $p'$ and replaces $x$'s priority with $\min(p, p')$.
- A DeleteMin operation finds the element with minimum priority in $Q$ and deletes and returns it.

If only comparisons between priorities are allowed, one can prove that either Insert or DeleteMin operations require logarithmic time. If the priorities are integers, and we can use their values as direct indices into an array, we can do better. In particular, assume the priorities are integers between 1 and $u$. Then this question asks you to describe a priority queue data structure and the implementation of the above operations on this data structure so that all operations except DeleteMin take constant time. A DeleteMin operation should take time proportional to $1 + p$, where $p$ is the priority of the returned element. Argue briefly that the operations are correct and achieve the required running time in the RAM model.