

Empirical Support for the Lower Bound GP Schema Theorem

Recap

- Principle observations which provided the basis for the Lower Bound GP Schema Theorem with 1-point Crossover Operator were [1],
 - Routed tree schema;
 - GA motivation for,
 - Don't care operator '=';
 - Point Mutation;
 - 1-point Crossover;
 - Order Operator $O(H) \equiv$ number of non '=' symbols;
 - Length $N(H) \equiv$ number of nodes in schema;
 - Defining Length $L(N) \equiv$ number of links in minimal sub-tree linking all non '=' nodes.
- Earlier derived the corresponding GP schema theorem,

$$E[m(H, t + 1)] \geq m(H, t) \frac{f(H, t)}{\bar{f}(t)} (1 - p_{mutate})^{O(H)} \left\{ 1 - p_{crossover} \left[p_{diff}(t) \left(1 - \frac{m(G(H), t) f(G(H), t)}{M \bar{f}(t)} \right) + \frac{L(H)}{N(H) - 1} \times \frac{m(G(H), t) f(G(H), t) - m(H, t) f(H, t)}{M \bar{f}(t)} \right] \right\}$$

- Where
 - $P_{diff}(t)$ was introduced to denote the conditional probability that schema H is disrupted by crossover AND second parent does not match the shape of schema H .
- Conjectures resulting from the above theorem, providing that $p_{mutation} = 0$,
 - FOR $t \rightarrow 0$ THEN (a) $P_{diff}(t) \rightarrow 1$ AND (b) $m(G(H)) \bar{f}(G(H)) \ll M \bar{f}(t)$
 - expect significant variation in schema shape;
 - FOR $t \rightarrow \infty$ THEN (a) $P_{diff}(t) \rightarrow 0$ AND (b) $m(G(H)) \bar{f}(G(H)) \approx M \bar{f}(t)$
 - Population dominated by same size & shape schema;
 - Convergence to specific schema H as $t \rightarrow \infty$;
 - In the limit, we have a GA schema theorem.

Experiment

- In order to identify empirical support for the above conjectures we need to be able to explicitly count the number of schema and enumerate their (average) fitness.
- Is this a problem?
 - Number of schema in program of ‘ n ’ nodes is 2^n .
- Constraints to problem domain,
 - 2 input Ex-OR function.

A	B	Z
0	0	0
0	1	1
1	0	1
1	1	0

- Constraints to individuals,

F = {AND, OR, NAND, NOR}		T = {A, B}
M = 50	generations = 50	Population initialisation: Grow
Depth Limit: 2 (3)	Corresponding Max schema count: 128 (32k)	$P_{crossover} = 0.35$

- Experimental framework.
 - Mutation is not of interest;
 - Consider case of following three experiments,
 - Selection alone;
 - Selection with 2 point crossover;
 - Selection with 1 point crossover;
 - 10 different population initialisations per experiment, but same set of 10 initialisations repeated across each experiment.

Selection Alone

- *Case 1* – Typical run statistics.
 - Mean Population Fitness;
 - Number of different Individuals/ Shapes/ Schema.
- Figure 1.

Parameter	$t = 0$	$t = 11$
Number of $G(H)$	4	1
Number of unique Programs	46	4
Number of unique H	2,892	≈ 400
Av(H per unique program)	62.8	≈ 100

- Question – how is it that schema continue to vary once $G = 1$?
- **Case 2** – Detailed view of specific schema instance.
 - Figure 2.
 - Hyperspace G_2 or (= (= = =) (= = =))
 - Hyperplane H_1 or (AND (= = A) (= = A))
 - Hyperplane H_2 or (AND (OR = =) (= = =))
 - Where $f(H_1) \approx f(H_2)$
- **Case 3** – Population/ Schema fitness: Multiple Trials
 - Figure 1(a)
 - Diversity and rate of decline in higher order schema, H (programs) \gg lower order schemas, G (hyperspaces).
 - Degree of selection in higher order schema higher on average.
 - Intuition,
 - higher order schema are more specific and therefore sensitive to fitness related content or “brittleness”.

Selection with 2 point crossover

- Each parent selects crossover points independent of “alignment.”
- Crossover points re-sampled until depth constraint satisfied.
- **Case 1** – Typical run statistics – Figure 5.
 - Mean Population Fitness;
 - Number of different Individuals/ Shapes/ Schema.
- **Case 3** – Population/ Schema fitness: Multiple Trials – Figure 1(b)
- Comments,
 - Brief initial phase in which program variation declines.
 - Apparent that crossover maintains a *constant* baseline of diversity.

- Many more schema at end of run than in case of selection alone.
- Crossover in this case appears to prevent the hyperspace competition from completing, thus maintaining hyperplane diversity at an exceptionally high level.

1-Point Crossover

- *Case 1* – Typical run statistics – Figure 8.
 - Mean Population Fitness;
 - Number of different Individuals/ Shapes/ Schema.
 - Competition for shape ends very quickly,
 - Single shape (hyperspace) at generation 5, with 10 unique programs;
 - Convergence at generation 19.
 - Independent hyperplane competition maintained for extensive period once shape converged.
- *Case 3* – Population/ Schema fitness: Multiple Trials – Figure 1(c)
 - Staggered nature of the competition always observed;
 - With respect to selection only,
 - Rate of diversity decay in 1-point crossover < selection alone;

Discussion

- 2-Point Crossover (with depth constraint) introduces significantly more schema disruption.
 - Actually prevented exploitation(!)
 - Total number of schema appear to perform a stochastic walk about some mean value.
- 1-Point Crossover
 - Total number of schema for ‘early’ generations \equiv 2-Point Crossover case;
 - Significantly faster decay/ convergence of hyperspace supports independent modelling of shape and schema by GP schema theorem.
 - Hyperspace always observed to converge to 1 whilst hyperplane competition continues implies that,
 - Competition between same size and shape programs takes place;
 - Increasing equivalent to GAs as $t \rightarrow \infty$.

Conclusion

- 2-Point Crossover (with depth constraint) appears to counteract building-block hypothesis;
- 1-Point Crossover appears to support building-block hypothesis;
- Multilevel nature of hyperspace/ hyperplane competition confirmed, and apparent (to different degree's) in each case;
- Role of mutation emphasized more with 1-Point Crossover;
- Genetic Drift plays a significant role when all individuals possess same/ similar fitness.

Reference

1. Poli R., Langdon W.B., (1997) "An Experiment Analysis of Schema Creation, Propagation and Disruption in Genetic Programming," Proceedings of the 7th International Conference on Genetic Programming. Bäck T. (ed), Morgan Kaufmann. Pp 18-25.