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Navigation with IMU/GPS/Digital Compass with Unscented Kalman Filter

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***Abstract*— Autonomous vehicle navigation with standard IMU and differential GPS has been widely used for aviation and military applications. Our research interesting is focused on using some low-cost off-the-shelf sensors, such as strap-down IMU, inexpensive single GPS receiver. In this paper, we present an autonomous vehicle navigation method by integrating the measurements of IMU, GPS, and digital compass. Two steps are adopted to overcome the low precision of the sensors. The first is to establish sophisticated dynamics models which consider Earth self rotation, measurement bias, and system noise. The second is to use a sigma Kalman filter for the system state estimation, which has higher accuracy compared with the extended Kalman filter. The method was evaluated by experimenting on a land vehicle equipped with IMU, GPS, and digital compass.**

Index Terms*— Sensor fusion. Navigation. Unscented Kalman filter. Estimation.

I. Introduction

Autonomous navigation vehicles, used for military missions, forest surveys, and ocean environment inspection, usually employ multiple sensors of various types. The sensors commonly used in these applications can be classified into two broad categories [4]: dead-reckoning sensors and external sensors. Generally dead-reckoning sensors are very robust, but accumulate errors with time. Therefore, they must be periodically reset by using information from external sensors. External sensors provide absolute information by making measurements from known landmarks. In our navigation platform, a low cost Inertial Measurement Unit (IMU) is used as a dead-reckoning sensor, while a Global Positioning System (GPS) receiver and a digital compass are used as external sensors for the outdoor navigation mission.

The rapidly expanding use of the Global Positioning System (GPS) enables commercial navigation devices to be more popular and attainable for the civil users. GPS provides absolute positioning information covering any part of the world during day and night. However, the signal quality from an adequate number of GPS satellites for the recipient is still critical in using GPS as navigation devices, such as under trees, inside buildings, in tunnels, between tall buildings, and under-water [5]. Therefore, for a reliable navigation system, another type of sensor, such as IMU, is needed.

The inertial navigation system (INS), originally developed in the mid 60s for Missile Guidance systems, measures the accelerations and rotations applied to a system's inertial frame of reference. These measurements are done via an inertial measurement unit (IMU) that consists of three linear accelerometers (devices that measure acceleration) and three rate gyroscopes (devices that measure angular rotation rate). An IMU system assembled from low-cost solid state components is always constructed in a *strap-down* configuration. The strap-down means that the gyroscopes and accelerometers are fixed to a common chassis and are not actively controlled on gimbals to align themselves in a prespecified direction [4]. The strap-down configuration makes the IMU widely accessible. But the tradeoff for a low cost IMU is its high noise and low accuracy. These drawbacks of the strap-down IMU can be overcome by careful filter design which is based on appropriate dynamic modeling.

In this paper, we present a general dynamic model for a strap-down IMU sensor for an autonomous navigation vehicle. The initial state of the IMU is calibrated by using a digital compass. And the IMU, GPS receiver, and digital compass are combined by using an unscented Kalman filter (UKF) [3] to obtain an optimal state of the vehicle.

II. Previous Work

The integration of an inertial measurement unit (IMU) with a global positioning system (GPS) receiver has been done for application areas in which attitude information is indispensable and rapid collection of geographic information is required. Because of its high price and government regulation, the high performance IMU is only used in military application and commercial airliners. Recent research efforts have been focused on using a low-cost strap-down IMU.

Farrell and Barth [1] introduced a very general method to establish an error process model for the INS/GPS navigation system. The error process model is obtained by using first order perturbation from the IMU dynamic models. A Kalman filter is used to estimate the errors in the IMU measurement.

In the low-cost IMU and GPS navigation system, the initial attitude of the IMU is very important. Nebot and Durrant-Whyte [4] presented a very clear and simple approach for the initial calibration and alignment of a low-cost IMU for land vehicle application based on the error process model, where two stable pendulum gyros are used to provide external attitude information for the IMU calibration. Salychev *et al.* [6] used external heading information to align the IMU. Sukkarieh [7] proposed the use of non-holonomic constraints, which describe the characteristics of the motion of land vehicles. which states that the motion of a wheeled vehicle on a surface is governed by two non-holonomic constraints.

The low cost IMU is peculiar in its weak stand-alone accuracy and poor run-to-run stability, which can result in large errors over short time intervals if their errors are not compensated [6]. Merwe and Wan [8] presented a dynamic process model which includes time varying bias terms. They used first-order Euler integration for system update and unscented Kalman filter (UKF) for system estimation. This method was successfully applied to helicopter's navigation. Huster [2] also used UKF for the relative position sensing by fusion monocular vision and inertial rate sensors.

The contribution of the paper is that we present a set of general dynamic models of IMU to describe the state of a robot. These dynamic equations are integrated by a fourth-order Runge-Kutta approach instead of a first order Euler integration as in [8]. UKF is used to fuse the data from GPS receiver and IMU, in which an external sensor, digital compass, is applied for the IMU's calibration.

The paper is organized as follows. In section III, we establish a process model and measurement model for the IMU and GPS integration system. In section IV, we introduce the unscented Kalman filter for the non-linear process model and measurement model, which has more accuracy then the extended Kalman filter. In section V, we address the issue for the implementation of the system. In section VI, we discuss the experiment results. In the last section, we present a summary and conclusion.

III. System Models

There are four reference frames related to a robot navigation, which are the inertial frame, earth frame, navigation frame, and body frame. The inertial frame (i-frame) is a reference frame in which Newton's laws of motion apply. All the inertial sensors make measurements relative to an inertial frame. The inertial coordinate system can take any point as its origin, and three mutually perpendicular directions as its axis. The earth frame (e-frame) has its origin fixed to the center of the earth. There are two different coordinate systems, rectangular and geodetic coordinate system, to describe the location of a point in the e-frame. The navigation frame (n-frame) is attached to a fixed point on the surface of the earth at some convenient point for local measurements. The body frame (b-frame) is rigidly attached to the vehicle of interest, usually at a fixed point such as the gravity center of the vehicle, which point is also the origin of the body coordinate system. These definition of all the frames is given in [1].

In this paper, we use a fixed navigation frame (n-frame) for the vehicle dynamics analysis. There are two issues which need to be clearly understood. The first is that the n-frame will not move with respect to the earth frame (e-frame). The second is that any movement related to the body frame (b-frame) must be projected to the n-frame.

A. Process Model

In the IMU/GPS/DigitalCompass supported vehicle navigation, we define a state vector x as follows.

$$x = \begin{pmatrix} p & v & q & b_a & b_\omega \end{pmatrix}^T \tag{1}$$

where $p = [x, y, z]^T$, $v = [v_n, v_e, v_d]^T$, and $q = [q_0, q_1, q_2, q_3]^T$ represent the position, velocity, and attitude in quaternion of the navigation vehicle in the n-frame. b_a is the IMU acceleration biases; and b_ω is the IMU gyro rate biases.

The measurements from an inertial sensor are based on the inertial frame (e-frame), and they must be transformed to the n-frame by the knowledge of attitude of the vehicle. We know all movement in the i-frame follows Newton's law, so the dynamic models for the vehicle navigation can be established below.

$$\begin{pmatrix} \dot{p} \\ \dot{v} \\ \dot{q} \\ \dot{b}_a \\ \dot{b}_\omega \end{pmatrix} = \begin{pmatrix} v \\ C_b^n \tilde{a}^b + g^n - (2\Omega_{ie}^n + \Omega_{en}^n) v_e^n \\ (w_{ib}^b - C_n^b(\omega_{ie}^n + \omega_{en}^n) - b_\omega - n_\omega) q \\ W_{b_a} \\ W_{b_\omega} \end{pmatrix} \tag{2}$$

From the property of IMU, despite of their quality, the acceleration and gyro rate output are known to be in error by a unknown slowly time-varying bias. Usually the turn-on bias is accurately known and accounted for in the IMU calibration. The residual time-varying bias error is modelled as a random-walk process [1]. Therefore, W_{b_a} and W_{b_ω} in Eq. (2) are white noise of acceleration and gyro rate, respectively, in the IMU.

In the land vehicle navigation, the vehicle's movement is in a limited geographic area. Therefore the fixed n-frame is sufficient. Otherwise the geodetic coordinate system needs to be used. For the fixed n-frame, the ω_{en}^n is a zeros vector. The ω_{ie}^n in Eq. (2) is the rotation rate of the e-frame with respect to the i-frame projected to the n-frame. This is defined as

$$\begin{aligned} \omega_{ie}^n &= C_e^n \omega_{ie}^e \\ &= \omega_e \begin{pmatrix} cos\lambda & 0 & -sin\lambda \end{pmatrix}^T \end{aligned} \tag{3}$$

where $\omega_e = 7.292115 \times 10^{-5} rad/s$ [1] is the rotation rate of the e-frame to the i-frame. λ is the latitude of the origin of the n-frame in the geodetic coordinate system. The ω_{ib}^b in Eq.(2) is the rotation rate measurement of gyros in IMU.

In Eq. (2), the C_b^n is the direction cosine matrix (DCM) from the b-frame to n-frame. Assuming the vehicle has an attitude which can be obtained by three successive rotation of angles ϕ, θ, and ψ around the x, y, and z axis, respectively.

The transformation matrix C_b^n is expressed by

$$C_b^n = \begin{pmatrix} c\theta c\psi & s\phi s\theta c\psi - c\phi s\psi & s\phi s\psi + c\phi s\theta c\phi \\ c\theta s\psi & c\phi c\psi + s\phi s\theta s\psi & c\phi s\theta s\psi - s\phi c\psi \\ -s\theta & s\phi c\theta & c\phi c\theta \end{pmatrix} \tag{4}$$

where s represents sin, and c represents cos. And g^n is the gravity vector in the n-frame, which is expressed by

$$g^n = \begin{pmatrix} 0 & 0 & g \end{pmatrix}^T \tag{5}$$

where g is the local gravity, which is decided by the coordinates in the geodetic coordinate system [1]. The $\tilde{a}^b$ in Eq.(2) is defined as

$$\tilde{a}^b = a_{ib}^b - b_a - n_a \tag{6}$$

where a_{ib}^b is the acceleration measurement from accelerameter in IMU, and n_a is the process noise. The Eq.(2) can be simply expressed as

$$\frac{d}{dt}x = f(x, a_{ib}^b, \omega_{ib}^b, n_p) \tag{7}$$

where n_p is the process noise, which is n_a or n_ω.

B. Measurement Model

The position obtained from GPS is considered as the measurement set in a Kalman filter. This can be expressed in the following formulation.

$$z_k = p_k + C_b^n r_{gps} + \nu_{gps} \tag{8}$$

where r_{gps} is the vector from the origin of the b-frame to the GPS mounting place. ν_{gps} is the GPS measurement noise, which is white with normal probability distribution $p(v) \sim N(0, R)$. The measurement noise matrix $R = diag(\sigma_x^2, \sigma_y^2, \sigma_z^2)$ can be calculated by statistics from a set of data which is obtained from the GPS in a fixed point in its working environment.

IV. Unscented Kalman Filter

A. Extended Kalman Filter

The extended Kalman filter (EKF) is a set of mathematical equations which uses an underlying process model to make an estimate of the current state of a system and corrects the estimate using any available sensor measurements. Using this predictor-corrector mechanism, it approximates an optimal estimate from the linearization of the process and measurement models [10]. Here is a brief introduction of the EKF:

Assuming the process has a state vector $x \in R^n$, but the process is now governed by the non-linear stochastic difference equation

$$x_k = f(x_{k-1}, w_{k-1}) \tag{9}$$

with a measurement $z \in R^m$ that is

$$z_k = h(x_k, v_k) \tag{10}$$

where the random variable w_k and v_k represent the process and measurement noise respectively. They are assumed to be independent of each other, white, and with normal probability distributions $p(w) \sim N(0, Q)$ and $p(v) \sim N(0, R)$. The system dynamic models $f(.)$ and $h(.)$ are assumed to be known.

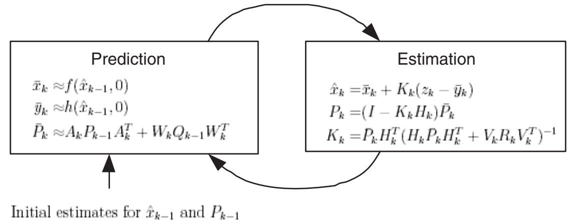

Fig. 1. A high level of the operation of the Extended Kalman filter. $\hat{x}_k$ and $\bar{x}_k$ represent estimate and predict of the state x at time step k, respectively.

Just like the basic Kalman filter, the extended Kalman filter is also carried out in two steps: prediction and estimation (Fig. 1). It is necessary to point out that a fundamental issue with the EKF is that the distributions (or densities in the continuous case) of the various random variables are no longer normal after undergoing their respective nonlinear transformations. The EKF is simply an *ad hoc* state estimation that only approximates the optimality of Bayes' rule by linearization [10].

From the above recursive steps, the prediction covariances $\bar{P}_k$ are determined by linearizing the system model, Eqs. (9) and (10), around the current estimate of the state and determining (approximating) the posterior covariance matrices analytically for the linear system. This is equivalent to applying the linear Kalman filter covariance update equations to the first-order linearization of the nonlinear system. Therefore, the EKF can be viewed as providing first-order approximated estimation, and these approximations can result in large errors in the estimates and even divergence of the filter.

B. Unscented Kalman Filter

UKF uses a deterministic sampling approach to capture the mean and covariance estimates with a minimal set of sample points, and it has 3rd order (Taylor series expansion) accuracy for Gaussian error distribution for any non-linear system [9], [3]; while EKF uses linearizing Jacobian matrix, which is a first order approximation. The Unscented Kalman filter (UKF) is claimed to have obvious advantage over EKF [3]. A brief overview of the UKF algorithm is presented in this section.

The unscented transformation (UT) is a method for calculating the statistics of a random variable which undergoes a nonlinear transformation. The n dimensional random variable x with mean $\bar{x}$ and covariance P_{xx} is approximated by $2n+1$ weighted points given by

$$\begin{aligned} \chi_0 &= \bar{x} \\ \chi_i &= \bar{x} + (\sqrt{(n+\kappa)P_{xx}})_i \qquad i = 1, \cdots, n \\ \chi_i &= \bar{x} - (\sqrt{(n+\kappa)P_{xx}})_i \qquad i = n+1, \cdots, 2n \end{aligned} \tag{11}$$

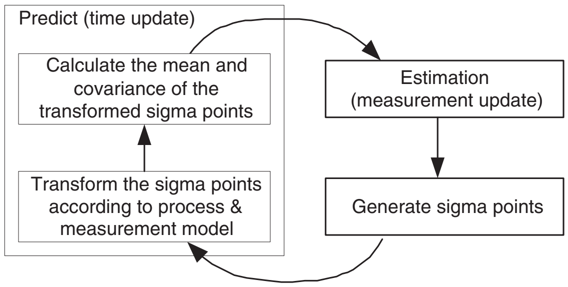

Fig. 2. A high level of the operation of the Unscented Kalman filter

$$\begin{aligned} W_0^m =& \kappa/(n+\kappa) \\ W_0^c =& \kappa/(n+\kappa) + (a - \alpha^2 + \beta) \\ W_i^c =& W_i^m = 1/[2(\kappa+n)] \qquad\qquad i = 1, \cdots, 2n \end{aligned} \tag{12}$$

where $\kappa = \alpha^2(n+\lambda) - n$ is a scaling parameter. λ determines the spread of the sigma points around $\bar{x}$ and is usually set to a small positive value. λ is a secondary scaling parameter which is usually set to 0, and β is a parameter used to incorporate any prior knowledge about the distribution of x. $(\sqrt{(n+\kappa)P_{xx}})_i$ is the ith row or column of the matrix square root of $((n+\kappa)P_{xx})$ and W_i is the weight which is associated with the ith point. These sigma points are propagated through the function

$$\mathcal{Y}_i = f(\chi_i) \qquad i = 0, \cdots, 2n \tag{13}$$

and the mean and covariance for $\mathcal{Y}$ are approximated using a weighted sample mean and covariance of the posterior sigma points,

$$\bar{y} = \sum_{i=0}^{2n} W_i^m \mathcal{Y}_i \tag{14}$$

$$P_{yy} = \sum_{i=0}^{2n} [\mathcal{Y}_i - \bar{y}][\mathcal{Y}_i - \bar{y}]^T \tag{15}$$

The unscented Kalman filter (UKF) can be implemented using UT by expanding the state space to include the noise component: $x_k^a = (x_k^T, w_k^T, v_k^T)^T$. The UKF can be summarized as follows [3], [9].

1. Initialization:

$$\hat{x}_0^a = \begin{pmatrix} \hat{x}_0^T & 0 & 0 \end{pmatrix}^T \tag{16}$$

$$P_0^a = \begin{pmatrix} P_0 & 0 & 0 \\ 0 & Q & 0 \\ 0 & 0 & R \end{pmatrix} \tag{17}$$

2. Iteration for each time step $k(\in 1, \cdots, \infty)$

a). Calculation the sigma points

$$\chi_{k-1}^a = \begin{pmatrix} \hat{x}_{k-1}^a & \hat{x}_{k-1}^a \pm \sqrt{(n+\kappa)P_{k-1}^a} \end{pmatrix} \tag{18}$$

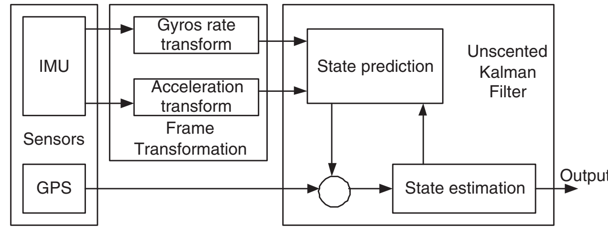

Fig. 3. Systems implementation digram

b). Time update:

$$\bar{\chi}_k^x = f(\chi_{k-1}^x, \chi_{k-1}^w) \tag{19}$$

$$\bar{x}_k = \sum_{i=0}^{2N} W_i^m \bar{\chi}_{i,k}^x \tag{20}$$

$$\bar{P}_k = \sum_{i=0}^{2N} W_i^c [\bar{\chi}_{i,k}^x - \hat{x}_{k-1}][\bar{\chi}_{i,k}^x - \hat{x}_{k-1}]^T \tag{21}$$

$$\bar{Y}_k = h(\bar{\chi}_k^x, \bar{\chi}_k^v) \tag{22}$$

$$\bar{y}_k = \sum_{i=0}^{2N} W_i^m \bar{Y}_k \tag{23}$$

c). Measurement update:

$$P_{y_k y_k} = \sum_{i=0}^{2N} W_i^c [\bar{Y}_{i,k} - \bar{y}_k][\bar{Y}_{i,k-1} - \bar{y}_k]^T \tag{24}$$

$$P_{x_k y_k} = \sum_{i=0}^{2N} W_i^c [\bar{\chi}_{i,k} - \bar{x}_k][\bar{Y}_{i,k} - \bar{y}_k]^T \tag{25}$$

$$K = P_{y_k y_k} P_{x_k y_k} \tag{26}$$

$$\hat{x}_k = \hat{x}_{k-1} + K(z_k - \bar{y}_k) \tag{27}$$

$$P_k = P_{k-1} - K P_{y_k y_k} K^T \tag{28}$$

The N in the previous equations is the dimension of the expended state space, which is equal to $N_x + N_w + N_v$. Where N_x is the dimension of the original state x_k; N_w and N_v are the dimension of noise w_k and v_k, respectively. In Eq.(17) Q is the process noise covariance, and R is the measurement noise covariance. W_i is the weight calculated by Eq.(12).

V. System Implementation

As soon as the process model and measurement model have been established, the Unscented Kalman filter procedure can be implemented directly. The system diagram is shown in Fig. 3. For our practical case, two issues must be addressed.

A. Sigma point propagation of UKF

The implementation of the unscented Kalman filter needs to create sigma points from a mean and covariance. Before creating the sigma points, the state vector has to be augmented to handle nonlinear process noise n_p and measurement noise ν_{gps}. Let the augmented states be $Y \in \Re^{N \times 1}$, $(N = N_x + N_n + N_{nu})$. Where N_x is the number of state variables, and N_n and N_{nu} are the dimension of process noise and the dimension of measurement noise, respectively. Then in any time step k

of the UKF, we can obtain a set of sigma points y_i ($i = 1 \cdots 2N+1$). These sigma points are transformed by using the dynamic process model (Eq. (7)) to obtain the points χ_i. Since the process model is nonlinear equations, we can assume the nonlinear function is defined as

$$\begin{aligned} \chi_i &= F(y_i, a^b, \omega_{ib}^b, n_p) \\ &= \int f(y_i, a^b, \omega_{ib}^b, n_p) dt \end{aligned} \tag{29}$$

where $i = 1 \cdots 2N+1$. For every point χ_i, we can implement 4th order Runge Kutta integration between the time step $k-1$ and k. The time interval is defined as ΔT. At time step $k-1$, we have $y(k-1)$. And at time step k, the IMU has provided the measurement $a_{ib}^b(k), \omega_{ib}^b(k)$. The $\chi_i(k)$ can be obtained using the algorithm 1.

Algorithm 1: 4th order Runge Kutta integration
Input: $y_i(k-1)$ and $a_{ib}^b(k), \omega_{ib}^b(k)$, n, and time ΔT
Output: predicted sigma point $\chi_i(k)$

$$\begin{aligned} k_1 &= \Delta T f(y_i(k-1), a_{ib}^b(k), \omega_{ib}^b(k)) \\ k_2 &= \Delta T f(y_i(k-1) + \tfrac{1}{2}\Delta T k_1, a_{ib}^b(k), \omega_{ib}^b(k)) \\ k_3 &= \Delta T f(y_i(k-1) + \tfrac{1}{2}\Delta T k_2, a_{ib}^b(k), \omega_{ib}^b(k)) \\ k_4 &= \Delta T f(y_i(k-1) + \Delta T k_3, a_{ib}^b(k), \omega_{ib}^b(k)) \\ \chi_i(k) &= y_i(k-1) + \tfrac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4) \end{aligned}$$

B. Synchronization with INS

Usually, measurement frequency in IMU is higher than that in GPS. And their measurement times will not be coincident (Fig. 4). In real time, linear extrapolation is used to obtain the IMU's predict position at the time the GPS gets its position by the following equation (Fig. 5)

$$p^{IMU}(t_{GPS}) = p^{IMU}(t_{k-1}) + \frac{p^{IMU}(t_{k-1}) - p^{IMU}(t_{k-2})}{t_{k-1} - t_{k-2}} (t_{GPS} - t_{k-1}) \tag{30}$$

where P^{IMU} means the predicted position of IMU. From Fig. 4, we know the IMU always predicts the vehicle state as soon as it gets measurement, and the navigation system starts to estimate its position when the GPS receives measurement data. The previous state prediction only based on the IMU can be updated based on the estimation.

VI. Experimental Results

In our field experiment, three sensors (an IMU-C300, a GPS, and a digital compass) are installed on a car with sun-roof. The IMU and digital compass are mounted at the center of the car, and the GPS is placed on the top of the car through the sun-roof window (Fig.6). During the experiment, we set the measurement frequency to 20Hz for the IMU and digital compass, and 1Hz for the GPS. The experiment was implemented on a parking lot of our campus. The raw measurement data for IMU is shown on Fig.7. Since the strap-down IMU is very sensitive to noise and environment, the measurement data are denoised with wavelet filter before they are used for vehicle state estimation. Fig.8 is the measurement data after denoising and initial alignment based on the digital compass measurement at the static state of the system.

The IMU is a dead reckoning sensor, and any noise and bias in the measurement of accelerometer in the IMU will cause second order deviation for its position estimate. And any noise and bias in the measurement of gyro in the IMU will change the estimate of the direction of vehicle greatly. So the measurements from a strap-down IMU can only be

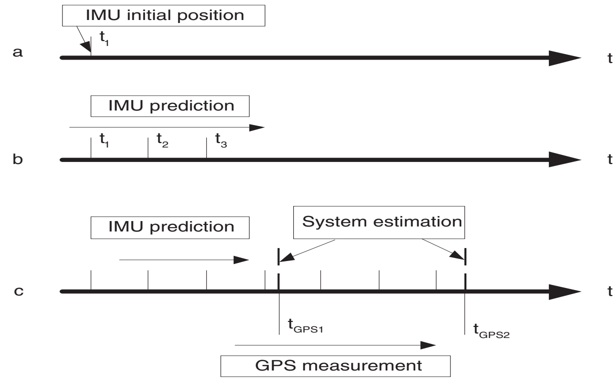

Fig. 4. IMU and GPS measurement time steps. a. IMU initial position obtained from calibration; b. system predict according to IMU measurement; c. system estimation when GPS measurement is received

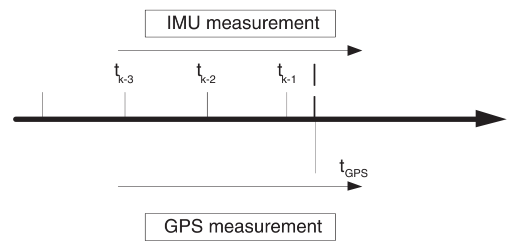

Fig. 5. IMU and GPS measurement time steps for linear extrapolation

Fig. 6. Experimental setup for land vehicle navigation with IMU, GPS and Digital Compass

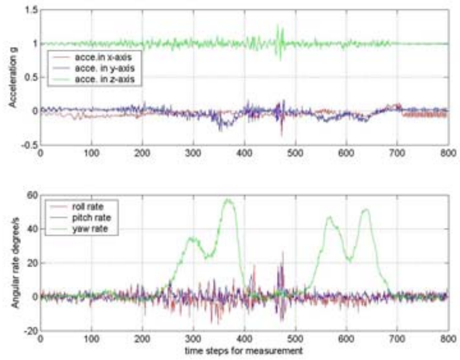

Fig. 7. Raw measurement data from IMU. The top is the accelerations from accelerameters in IMU, and the bottom is the rotation rate from gyros in IMU

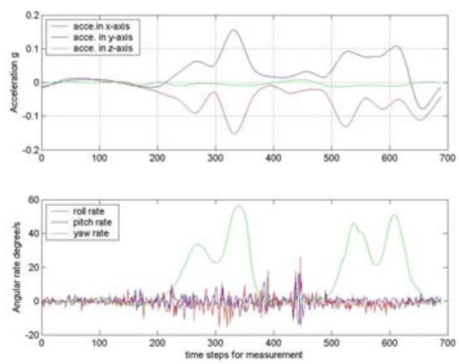

Fig. 8. The IMU measurements after denoising and initial alignment. The top is the accelerations, and the bottom is the rotation rate

reliable for a very short time. In Fig. 9, the fusion of IMU and GPS provides a continuously vehicle trajectory. Although we can not obtain the ground truth of the trajectory, the result is reasonable and consistent with respect to the dimensions of the parking lot.

It must be pointed out that the strap-down IMU is very sensitive to its working environment. Careful setup and initial system alignment during the experiment is very important.

VII. Conclusion

We present a vehicle navigation method by integrating the measurements of IMU, GPS, and digital compass. The measurement of an inertial sensor is based on the i-frame, and the measurement of a GPS receiver is based on the e-frame, but the vehicle navigation is based on a fixed local n-frame. So the dynamic models for the system process include the associated transformations from the i-frame and e- frame to the n-frame. The non-linear process models are integrated with fourth order Runge Kutta method. And then the system estimation is implemented by using a sigma Kalman filter which has higher calculation accuracy compared with an extended Kalman filter. This method was evaluated by the

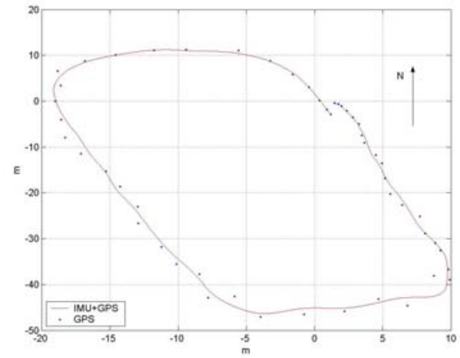

Fig. 9. The trajectory estimated from our field experiment

experimentation of a land vehicle equipped with IMU, GPS, and digital compass on a parking lot.

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