Problems Assignment 1

1.1 Consider the problem of adding n numbers. Assume that one person can add two numbers in time t_c. How long will a person take to add n numbers?

Now assume that eight people are available for adding these n numbers and that it is possible to divide the list into eight parts. The eight people have their own pencils and paper (on which to perform additions), are equally skilled, and can add two numbers in time t_c. Furthermore, a person can pass on the result of an addition (in the form of a single number) to the person sitting next to him or her in time t_w. How long will it take to add n numbers in the following scenarios:

(a) All eight people are sitting in a circle.

(b) The eight people are sitting in two rows of four people each.

1.2 Assume the scenario described in Problem 1.1. If you had the liberty of seating the people in any acceptable configuration, how would you seat them so as to minimize the time taken to add the list of numbers? (An acceptable configuration is defined as follows: Assume that the adders are vertices of a graph and that *neighborhood* is defined by the edges between the vertices. Any graph that can be drawn on a piece of paper and that has no intersecting edges represents an acceptable configuration.) What is the time taken by your configuration?

1.3 Consider again the problem of adding n numbers. Assume that one person takes time $t_c(n-1)$ to add these numbers. Is it possible for p people to add this list in time less than $t_c(n-1)/p$? Justify your answer.

1.4 Consider again the scenario of Problem 1.1, but assume that all eight people are adding the numbers standing at a blackboard. Each person can see results from the other person's calculations as they are completed (that is, instantaneously). How long would the eight people take to add the n numbers in this case?

1.5 Answer Problem 1.3 in the context of the scenario presented in Problem 1.4. Is your answer different for this scenario? If so, what specific change resulted in a different answer?

2.2 Consider an EREW PRAM with p processors and m memory locations. We can emulate this model on a p-processor message-passing parallel computer in which each processor has m/p memory locations. Let t be the run time of an algorithm on a p-processor EREW PRAM model. Give an upper bound on the run time of this algorithm on the following architectures:

(a) a p-processor ring

(b) a p-processor mesh

(c) a p-processor hypercube

2.6 A cycle in a graph is defined as a path originating and terminating at the same node. The length of a cycle is the number of edges in the cycle. Show that there are no odd-length cycles in a d-dimensional hypercube.

2.7 The labels in a d-dimensional hypercube use d bits. Fixing any k of these bits, show that processors whose labels differ in the remaining $d-k$ bit positions form a $(d-k)$-dimensional subcube composed of $2^{(d-k)}$ processors.

2.11 [Lei92] A ***mesh of trees*** is a network that imposes a tree interconnection on a grid of processors. A $\sqrt{p} \times \sqrt{p}$ mesh of trees is constructed as follows. Starting with a $\sqrt{p} \times \sqrt{p}$ grid of processors a complete binary tree is imposed on each row of the grid. Then a complete binary tree is imposed on each column of the grid. Figure 2.30 illustrates the construction of a 4×4 mesh of trees. Assume that the nodes at intermediate levels are switching elements. Determine the bisection width, diameter, and total number of switching elements in a $\sqrt{p} \times \sqrt{p}$ mesh of processors.

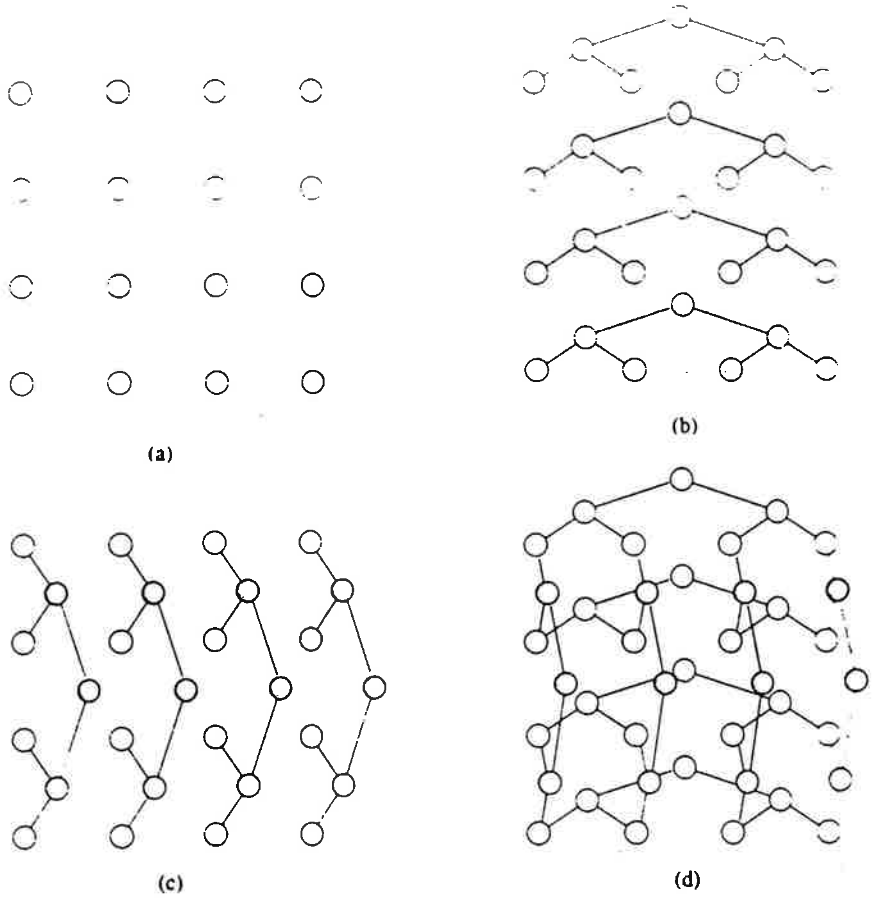

Figure 2.30 The construction of a 4×4 mesh of trees: (a) a 4×4 processor grid, (b) complete binary trees imposed over individual rows, (c) complete binary trees imposed over each column, and (c) the complete 4×4 mesh of trees.

Bonus Questions

2.14 [Lei92] One of the drawbacks of a hypercube-connected network is that different wires in the network are of different lengths. This implies that data takes different times to traverse different communication links. It appears that two-dimensional mesh networks with wraparound connections suffer from this drawback too. However, it is possible to fabricate a two-dimensional wraparound mesh using wires of fixed length. Illustrate this layout by drawing such a 4×4 wraparound mesh.

3) Describe in detail an efficient algorithm that sorts n distinct elements on a ~~hypercube of size~~ p=n^2. Hints: Find the rank of each item them move it to that position.
Think of the hypercube as a n x n mesh where each row and column is a sub-hypercube.