Faculty of Computer Science, Dalhousie University *7-Nov-2023*

CSCI 4152/6509 — Natural Language Processing

Lecture 19: Examples with Message-passing Algorithms

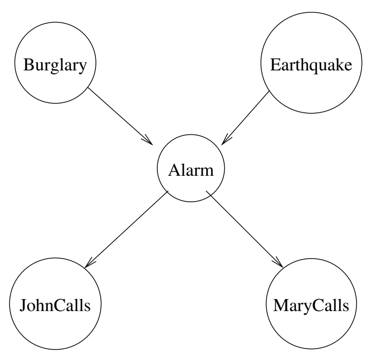
Location: Rowe 1011 Instructor: Vlado Keselj
Time: 16:05 – 17:25

Previous Lecture

- Message-passing
  1. Isolated factor node to variable node
  2. Isolated variable node to factor node
  3. General factor node to variable node
  4. General variable node to factor node
- Inference tasks using message passing
  1. Marginalization with one variable
  2. Marginalization with multiple variables
  3. Conditioning with one variable
  4. Conditioning with multiple variables
  5. Completion in general

16.4 Message-Passing Inference Algorithm: Burglar-Earthquake Example

In this example we use the previously given Burglar-Earthquake Bayesian Network:



The given tables are:

B	$\mathrm{P}(B)$
T	0.001
F	0.999

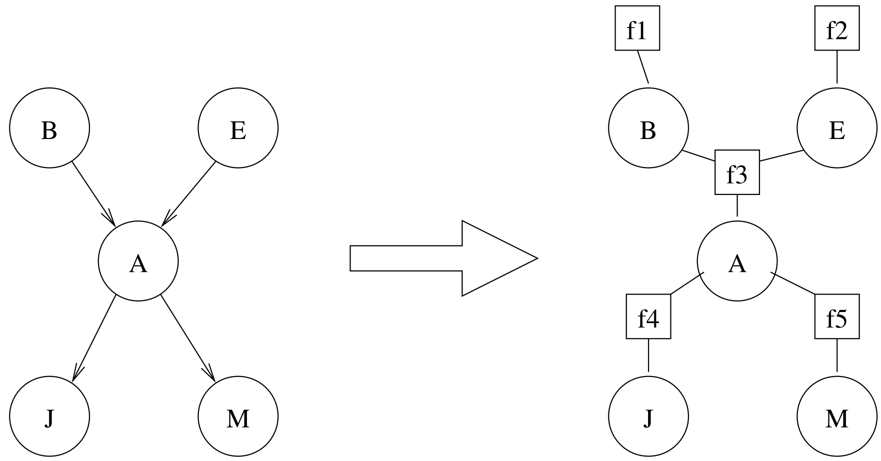
E	$\mathrm{P}(E)$
T	0.002
F	0.998

B	E	A	$\mathrm{P}(A\|B,E)$
T	T	T	0.95
T	T	F	0.05
T	F	T	0.94
T	F	F	0.06
F	T	T	0.29
F	T	F	0.71
F	F	T	0.001
F	F	F	0.999

A	J	$\mathrm{P}(J\|A)$
T	T	0.90
T	F	0.10
F	T	0.05
F	F	0.95

A	M	$\mathrm{P}(M\|A)$
T	T	0.70
T	F	0.30
F	T	0.01
F	F	0.99

Our first step is to translate this network into a factor graph:



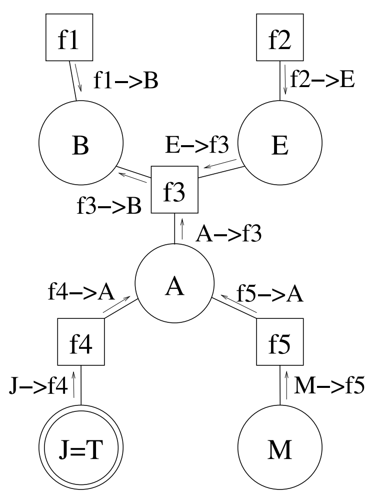
The function nodes correspond to conditional probabilities in the following way: $f_1 \sim \mathrm{P}(B)$, $f_2 \sim \mathrm{P}(E)$, $f_3 \sim \mathrm{P}(A|B,E)$, $f_4 \sim \mathrm{P}(J|A)$, and $f_5 \sim \mathrm{P}(M|A)$.

Burglar-Earthquake Example Problem

- John called, probability that Burglar is in the house
- $P(B = T|J = T) =?$
- Conditioning with one variable

Problem: Calculate the probability that a burglar is in the house, if we know that John has called.

We "hard-wire" the variable J to the value T, and analyze which messages we need to compute:



The messages are calculated in the following way:

$f_1 \to B$ is simple table copy of $\mathrm{P}(B)$:

B	$f_1 \to B$
T	0.001
F	0.999

Similarly:

E	$f_2 \to E$
T	0.002
F	0.998

E	$E \to f_3$
T	0.002
F	0.998

J is "hardwired"

to T (observed evidence) so we get:

J	$J \to f_4$
T	1
F	0

M is not "hardwired":

M	$M \to f_5$
T	1
F	1

Calculation of the remaining messages requires a bit more calculations:

$f_4 \to A$				
A	J	$J \to f_4$	f_4	
$A = T$	T	1	$\cdot 0.90$	$= 0.9$
	F	0	$\cdot 0.10$	$= 0$
			Σ	$= 0.9$
$A = F$	T	1	$\cdot 0.05$	$= 0.05$
	F	0	$\cdot 0.95$	$= 0$
			Σ	$= 0.05$

$f_5 \to A$				
A	M	$M \to f_5$	f_5	
$A = T$	T	1	$\cdot 0.70$	$= 0.7$
	F	1	$\cdot 0.30$	$= 0.3$
			Σ	$= 1$
$A = F$	T	1	$\cdot 0.01$	$= 0.01$
	F	1	$\cdot 0.99$	$= 0.99$
			Σ	$= 1$

Hence the messages are:

A	$f_4 \to A$
T	0.9
F	0.05

and

A	$f_5 \to A$
T	1
F	1

. The message $A \to f_3$ is obtained by component-wise multiplication of messages coming into A:

A	$A \to f_3$
T	0.9
F	0.05

Finally, we compute the message $f_3 \to B$:

$f_3 \to B$						
B	E	A	$E \to f_3$	$A \to f_3$	f_3	
$B = T$	T	T	0.002	$\cdot 0.9$	$\cdot 0.95$	$= 0.00171$
	T	F	0.002	$\cdot 0.05$	$\cdot 0.05$	$= 0.000005$
	F	T	0.998	$\cdot 0.9$	$\cdot 0.94$	$= 0.844308$
	F	F	0.998	$\cdot 0.05$	$\cdot 0.06$	$= 0.002994$
					Σ	$= 0.849017$

$f_3 \to B$						
B	E	A	$E \to f_3$	$A \to f_3$	f_3	
$B = F$	T	T	0.002	$\cdot 0.9$	$\cdot 0.29$	$= 0.000522$
	T	F	0.002	$\cdot 0.05$	$\cdot 0.71$	$= 0.000071$
	F	T	0.998	$\cdot 0.9$	$\cdot 0.001$	$= 0.0008982$
	F	F	0.998	$\cdot 0.05$	$\cdot 0.999$	$= 0.0498501$
					Σ	$= 0.0513413$

Hence, the message $f_3 \rightarrow B$ is:

B	$f_3 \rightarrow B$
T	0.849017
F	0.0513413

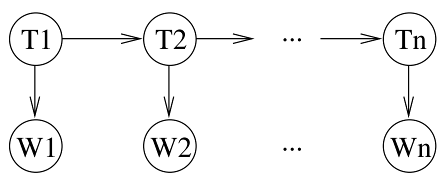
Final Calculation $P(B = T|J = T)$

Now, we can compute $\mathrm{P}(B = T|J = T)$ by multiplying component-wise the messages arriving at B, and by normalizing the result:

$$\begin{aligned} P(B = T|J = T) &= \frac{f_1 \rightarrow B(T) \cdot f_3 \rightarrow B(T)}{f_1 \rightarrow B(T) \cdot f_3 \rightarrow B(T) + f_1 \rightarrow B(F) \cdot f_3 \rightarrow B(F)} \\ &= \frac{0.001 \cdot 0.849017}{0.001 \cdot 0.849017 + 0.999 \cdot 0.513413} = 0.01628373 \end{aligned}$$

16.5 Message Passing Algorithm: POS Tagging Example

The HMM tagging using message passing would work as follows:



Training data:

```
swat V flies N like P ants N
time N flies V like P an D arrow N
```

Trained HMM Model:

T_1	$\mathrm{P}(T_1)$
N	0.5
V	0.5

,

T_{i-1}	T_i	$\mathrm{P}(T_i \mid T_{i-1})$
D	N	1
N	P	0.5
N	V	0.5
P	D	0.5
P	N	0.5
V	N	0.5
V	P	0.5

and

T_i	W_i	$\mathrm{P}(W_i \mid T_i)$
D	an	$2/3 \approx 0.666666667$
D	$\star$	$1/3 \approx 0.333333333$
N	ants	$2/9 \approx 0.222222222$
N	arrow	$2/9 \approx 0.222222222$
N	flies	$2/9 \approx 0.222222222$
N	time	$2/9 \approx 0.222222222$
N	$\star$	$1/9 \approx 0.111111111$
P	like	0.8
P	$\star$	0.2
V	flies	0.4
V	swat	0.4
V	$\star$	0.2

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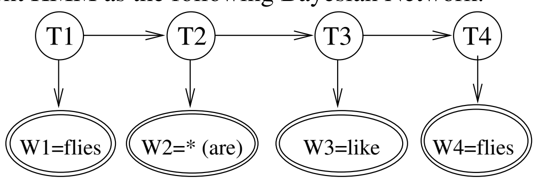
Tagging Example

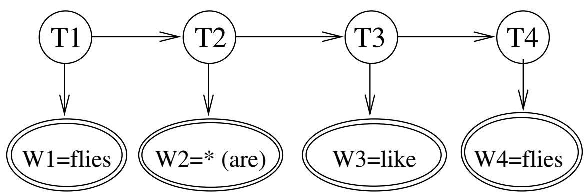
Slide notes:

Tagging Example

- Example: "flies are like flies"
- Represent HMM as the following Bayesian Network:



Let us again use the example sentence "flies are like flies", which we used in a previous example with HMM. First, we will represent HMM configuration as a Bayesian Network with observable variables "hard-wired" to their values, as follows:

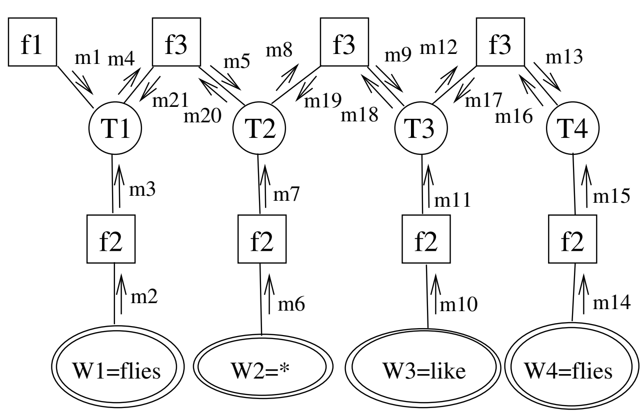


Slide notes:

POS Tagging as Message Passing

- Solving a completion problem
- Algorithm steps:
  - Create a factor graph
  - Hard-wire output variables
  - Use message passing with maximization
  - Find maximum-likely completion
- We will calculate only necessary messages

The corresponding factor graph is:



The messages are calculated as follows:

T_1	m_1
D	0
N	0.5
P	0
V	0.5

, and

W_1	m_2
flies	1
an	0
*	0
⋮	0

.

Calculation of m_3 is done as follows:

m_3			
$T_1 = D$	$W_1 =$ flies:	$1 \cdot 0$	$= 0$
	$W_1 =$ an:	$0 \cdot \frac{2}{3}$	$= 0$
	$W_1 =$ ⋮	⋮	$= 0$
			max:0
$T_1 = N$	$W_1 =$ flies	$: 1 \cdot \frac{2}{9}$	$= \frac{2}{9}$
	$W_1 =$ an	$: 0 \cdot \frac{1}{9}$	$= 0$
			max:2/9
⋮			

and we obtain

T_1	m_3
D	0
N	$2/9$
P	0
V	0.4

.

The other messages are:

T_1	$m_4 (= m_1 \cdot m_3)$
D	$0 \cdot 0 = 0$
N	$0.5 \cdot 2/9 = 1/9$
P	$0 \cdot 0 = 0$
V	$0.5 \cdot 0.4 = 0.2$

T_2	m_5
D	0
N	0.1
P	0.1
V	$1/18$

m_5 is calculated as follows:

m_5	$m_4 \cdot f_3$	
$T_2 = D$	$T_1 = D : 0 \cdot 0$	$= 0$
	$T_1 = N : \frac{1}{9} \cdot 0$	$= 0$
	$T_1 = P : 0 \cdot 0.5$	$= 0$
	$T_1 = V : 0.2 \cdot 0$	$= 0$
		max:0

m_5	$m_4 \cdot f_3$	
$T_2 = N$	$T_1 = D : 0 \cdot 1$	$= 0$
	$T_1 = N : \frac{1}{9} \cdot 0$	$= 0$
	$T_1 = P : 0 \cdot 0.5$	$= 0$
	$T_1 = V : 0.2 \cdot 0.5$	$= 0.1$
		max:0.1

m_5	$m_4 \cdot f_3$	
$T_2 = P$	$T_1 = D : 0 \cdot 0$	$= 0$
	$T_1 = N : \frac{1}{9} \cdot 0.5$	$= 1/18$
	$T_1 = P : 0 \cdot 0$	$= 0$
	$T_1 = V : 0.2 \cdot 0.5$	$= 0.1$
		max:0.1

m_5	$m_4 \cdot f_3$	
$T_2 = V$	$T_1 = D : 0 \cdot 0$	$= 0$
	$T_1 = N : \frac{1}{9} \cdot 0.5$	$= 1/18$
	$T_1 = P : 0 \cdot 0$	$= 0$
	$T_1 = V : 0.2 \cdot 0$	$= 0$
		max:$1/18$

We continue calculating:

W_2	m_6
flies	0
an	0
*	1
⋮	0

,

T_2	m_7
D	$1/3$
N	$1/9$
P	0.2
V	0.2

,

T_2	$m_8 (= m_5 \cdot m_7)$
D	$0 \cdot \frac{1}{3} = 0$
N	$0.1 \cdot \frac{1}{9} = 1/90$
P	$0.1 \cdot 0.2 = 0.02$
V	$\frac{1}{18} \cdot 0.2 = 1/90$

.

To calculate m_9, we have the following intermediate calculations:

m_9	$m_8 \cdot f_3$	
$T_3 = D$	$T_2 = D : 0 \cdot 0$	$= 0$
	$T_2 = N : \frac{1}{90} \cdot 0$	$= 0$
	$T_2 = P : \frac{1}{50} \cdot 0.5$	$= 0.01$
	$T_2 = V : \frac{1}{90} \cdot 0$	$= 0$
		max:0.01

m_9	$m_8 \cdot f_3$	
$T_3 = N$	$T_2 = D : 0 \cdot 1$	$= 0$
	$T_2 = N : \frac{1}{90} \cdot 0$	$= 0$
	$T_2 = P : \frac{1}{50} \cdot 0.5$	$= 0.01$
	$T_2 = V : \frac{1}{90} \cdot 0.5$	$= 1/180$
		max:0.01

m_9	$m_8 \cdot f_3$	
$T_3 = P$	$T_2 = D : 0 \cdot 0$	$= 0$
	$T_2 = N : \frac{1}{90} \cdot 0.5$	$= 1/180$
	$T_2 = P : \frac{1}{50} \cdot 0$	$= 0$
	$T_2 = V : \frac{1}{90} \cdot 0.5$	$= 1/180$
		max:$1/180$

m_9	$m_8 \cdot f_3$	
$T_3 = V$	$T_2 = D : 0 \cdot 0$	$= 0$
	$T_2 = N : \frac{1}{90} \cdot 0.5$	$= 1/180$
	$T_2 = P : \frac{1}{50} \cdot 0$	$= 0$
	$T_2 = V : \frac{1}{90} \cdot 0$	$= 0$
		max:$1/180$

and we obtain:

T_3	m_9
D	0.01
N	0.01
P	$1/180$
V	$1/180$

. Then,

W_3	m_{10}
like	1
$\vdots$	0

,

T_3	m_{11}
D	0
N	0
P	0.8
V	0

,

T_3	$m_{12} (= m_9 \cdot m_{11})$
D	$0.01 \cdot 0 = 0$
N	$0.01 \cdot 0 = 0$
P	$\frac{1}{180} \cdot 0.8 = 1/225$
V	$\frac{1}{180} \cdot 0 = 0$

.

To calculate m_{13}, we have the following intermediate calculations:

m_{13}	$m_{12} \cdot f_3$	
$T_4 = D$	$T_3 = D : 0 \cdot 0$	$= 0$
	$T_3 = N : 0 \cdot 0$	$= 0$
	$T_3 = P : \frac{1}{225} \cdot 0.5$	$= 1/450$
	$T_3 = V : 0 \cdot 0$	$= 0$
		max:$1/450$

m_{13}	$m_{12} \cdot f_3$	
$T_4 = N$	$T_3 = D : 0 \cdot 1$	$= 0$
	$T_3 = N : 0 \cdot 0$	$= 0$
	$T_3 = P : \frac{1}{225} \cdot 0.5$	$= 1/450$
	$T_3 = V : 0 \cdot 0.5$	$= 0$
		max:$1/450$

m_{13}	$m_{12} \cdot f_3$	
$T_4 = P$	$T_3 = D : 0 \cdot 0$	$= 0$
	$T_3 = N : 0 \cdot 0.5$	$= 0$
	$T_3 = P : \frac{1}{225} \cdot 0$	$= 0$
	$T_3 = V : 0 \cdot 0.5$	$= 0$
		max:0

m_{13}	$m_{12} \cdot f_3$	
$T_4 = V$	$T_3 = D : 0 \cdot 0$	$= 0$
	$T_3 = N : 0 \cdot 0.5$	$= 0$
	$T_3 = P : \frac{1}{225} \cdot 0$	$= 0$
	$T_3 = V : 0 \cdot 0$	$= 0$
		max:0

and we obtain:

T_4	m_{13}
D	$1/450$
N	$1/450$
P	0
V	0

. Then,

W_4	m_{14}
flies	1
$\vdots$	0

, and

T_4	m_{15}
D	0
N	$2/9$
P	0
V	0.4

.

To maximize the product of probabilities of T_4 we calculate:

T_4	$m_{13} \cdot m_{15}$	
D	$\frac{1}{450} \cdot$	$= 0$
N	$\frac{1}{450} \cdot \frac{2}{9}$	$= 1/2025$
P	$0 \cdot 0$	$= 0$
V	$0 \cdot 0.4$	$= 0$

and we obtain $\boxed{T_4^* = N}$, which we use in further messages, as a "hard-wired" value. We calculate

T_4	m_{16}
D	0
N	$2/9$
P	0
V	0

, and for m_{17} use only $T_4 = N$ in $m_{16} \cdot f_3$:

$m_{16} \cdot f_3$	
$\frac{2}{9} \cdot 1$	$= 2/9$
$\frac{2}{9} \cdot 0$	$= 0$
$\frac{2}{9} \cdot 0.5$	$= 1/9$
$\frac{2}{9} \cdot 0.5$	$= 1/9$

, and we obtain:

T_3	m_{17}
D	$2/9$
N	0
P	$1/9$
V	$1/9$

.

To find optimal T_3 we calculate:

T_3	$m_9 \cdot m_{11} \cdot m_{17}$	
D	$0.01 \cdot 0 \cdot \frac{2}{9}$	$= 0$
N	$0.01 \cdot 0 \cdot 0$	$= 0$
P	$\frac{1}{180} \cdot 0.8 \cdot \frac{1}{9}$	$= 1/2025$
V	$\frac{1}{180} \cdot 0 \cdot \frac{1}{9}$	$= 0$

and we obtain: $\boxed{T_3^* = P}$

Then,

T_3	$m_{18} = m_{17} \cdot m_{11}$
D	0
N	0
P	$\frac{1}{9} \cdot 0.8 = 4/45$
V	0

,

T_2	$m_{19} = m_{18} \cdot f_3$ for $T_3 = P$
D	$\frac{4}{45} \cdot 0 = 0$
N	$\frac{4}{45} \cdot \frac{1}{2} = 2/45$
P	$\frac{4}{45} \cdot 0 = 0$
V	$\frac{4}{45} \cdot \frac{1}{2} = 2/45$

.

To find optimal T_2 we calculate:

T_2	$m_{19} \cdot m_5 \cdot m_7$	
D	$0 \cdot 0 \cdot \frac{1}{3}$	$= 0$
N	$\frac{2}{45} \cdot 0.1 \cdot \frac{1}{9}$	$= 1/2025$
P	$0 \cdot 0.1 \cdot 0.2$	$= 0$
V	$\frac{2}{45} \cdot \frac{1}{18} \cdot 0.2$	$= 1/2025$

and we can choose either N or V. Let us choose $\boxed{T_2^* = V}$.

T_2	$m_{20} = m_7 \cdot m_{19}$
D	0
N	0
P	0
V	$0.2 \cdot \frac{2}{45} = 2/225$

,

T_1	$m_{21} = m_{20} \cdot f_3$ for $T_2 = V$
D	$\frac{2}{225} \cdot 0 = 0$
N	$\frac{2}{225} \cdot \frac{1}{2} = 1/225$
P	$\frac{2}{225} \cdot 0 = 0$
V	$\frac{2}{225} \cdot 0 = 0$

.

To find optimal T_1 we calculate:

T_1	$m_1 \cdot m_3 \cdot m_{21}$	
D	$0 \cdot 0 \cdot 0$	$= 0$
N	$0.5 \cdot \frac{2}{9} \cdot \frac{1}{225}$	$= 1/2025$
P	$0 \cdot 0 \cdot 0$	$= 0$
V	$0.5 \cdot 0.4 \cdot 0$	$= 0$

and we obtain $\boxed{T_1^* = N}$.

To summarize, the most probable values of unknown variables T_1, T_2, T_3, and T_4 are:

$$\boxed{T_1^* = N \quad T_2^* = V \quad T_3^* = P \quad T_4^* = N}$$